Cooperative Diversity of Spectrum Sensing for Cognitive Radio Systems

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***Abstract*—Spectrum sensing is a crucial issue in cognitive radio networks. To improve the sensing performance, cooperation among the secondary users can be utilized to collect space diversity. In this paper, we quantify the diversity order for various cooperative spectrum sensing strategies. We first notice that, in a spectrum sensing problem, diversity should reflect both the false alarm and missed detection behaviors since they respectively capture the efficiency and reliability of the overall cognitive system. Hence, we consider both false alarm and missed detection probabilities individually and jointly via the average error probability. With the knowledge of both the noise strength and the primary user signal strength, the threshold of the Neyman-Pearson detector can be varied to control the system performance in desirable ways. Interesting tradeoffs between system efficiency and reliability are found in various cooperative strategies and analytical results are presented to guide practical system designs with different preferences.**

***Index Terms*—Cognitive radio systems, cooperative diversity, spectrum sensing, system efficiency and reliability tradeoff.**

I. Introduction

THE explosive development of wireless services makes the spectrum a scarce resource. Although the spectrum is almost fully assigned to various licensed wireless users, its actual utilization is not quite efficient (see, e.g., [4]). To address such inefficiency, cognitive radio systems [6] were recently proposed as a means of filling the spectrum vacancy in time or space [9]. In cognitive radio systems, the unlicensed wireless users (a.k.a. secondary users) take chances to access the spectrum (temporarily or spatially) released by the licensed users (a.k.a. primary users) so that the spectrum access is dynamic and somewhat opportunistic [13]. To realize this, the first step is the finding of such opportunities in the primary users' spectrum usage; that is, the so-termed spectrum sensing.

Among existing works on spectrum sensing, some focus on algorithms to improve a single-user's sensing performance by utilizing some side information (see, e.g., [1], [5]). Nonetheless, the single-user spectrum sensing still consists of the system performance bottleneck due to the harsh wireless channel encountering fading and shadowing effects as well as the noise uncertainty of the device [10]. To this end, cooperative spectrum sensing by multiple secondary users can significantly improve the sensing performance. Hence, this has become the focus of most ongoing research (see, e.g., [8] and [12]). However, while *diversity* has been well acknowledged as the intuitive benefit of cooperative sensing, its rigorous meaning in this setup has remained largely unexplored. In this paper, we will quantitatively determine the diversity order in various cooperative spectrum sensing schemes.

Diversity has been widely adopted as a fundamental performance indicator in communication systems, where it is defined and quantified in terms of the signal-to-noise ratio (SNR-)dependent behavior of the bit error rate (BER) for symbol detection [11, Ch. 3] or the outage probability of the mutual information [11, Ch. 5]. This concept was recently extended to the context of cooperative estimation in wireless sensor networks [2]. Therein, diversity refers to the SNR-dependent behavior of the outage probability that the estimation variance exceeds a predefined value. In [12], the cooperative spectrum sensing process is considered by emphasizing the false alarm probability while fixing the missed detection probability. However, in the spectrum sensing problem, this does not sufficiently and appropriately facilitate the quantification of diversity.

Unlike traditional detection problems where focusing on either the false alarm or missed detection probabilities while fixing the other is a rather common exercise, doing so in a spectrum sensing problem will risk unbalanced treatment between the system *efficiency* and *reliability*. On the one hand, false alarm probability is of critical importance because the whole purpose of cognitive radio is to maximally utilize the spectrum vacancies, while false alarms lead to undetected spectrum holes and can significantly reduce the efficiency of such usage. On the other hand, missed detections lead to deteriorated "cognition" level and give rise to unexpected interference from the secondary users to the primary ones. In short, false alarm and missed detection probabilities respectively capture the *efficiency* and *reliability* of the overall cognitive system. Hence, diversity measure of cooperative sensing performance should fairly account for both probabilities. In this paper, we consider the false alarm and missed detection probabilities both individually and jointly in terms of the average error probability which balances between the system efficiency and reliability.

This new perspective accounting for both efficiency and reliability makes our work very unique with respect to existing ones such as [12]. First, with this perspective, the threshold of the energy detector can be adjusted to improve both performances simultaneously. Second, our study better reflects the nature of the spectrum sensing problem by quantitatively capturing the tradeoff between the efficiency and reliability in closed form.

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This tradeoff has never been observed or documented before due to the biased emphasis towards specific performance measures. Third, for the multiuser sensing with hard information fusion, the local detection strategy and the fusion detection strategy can be jointly optimized based on the tradeoff relationship established in our analysis.

Our technical contributions are summarized as follows. i) We derive the optimum detection thresholds by minimizing the average error probability in both noncooperative single-user and cooperative multiuser sensing scenarios. The diversity orders of all three probabilities are then quantified under the optimum thresholds. We also prove that such thresholds lead to the maximum diversity order in both sensing scenarios. ii) We consider two cooperative strategies, namely multiuser sensing with soft information fusion and hard information fusion. The former provides a theoretical bound on the diversity orders and error probability performance in an ideal cooperative sensing setup; whereas the latter leads to practical fusion and decision rules together with their respective quantified diversity orders. iii) We investigate the tradeoff between the system efficiency (via false alarm probability) and reliability (via missed detection probability) and present analytical results to guide practical system designs with differing preference. iv) Depending on whether the secondary users have the knowledge of the number of cooperative users, we find that the optimal hard fusion rules are respectively the majority-fusion rule and the OR-fusion rule. v) We verify the benefit of cooperative sensing and compare the performances of majority-fusion and OR-fusion rules at low SNR.

The signal model, cooperation strategies and the performance metrics with diversity definition will be given in Section II. The diversity orders of the single-user spectrum sensing will be analyzed in Section III, followed by various multiuser cases in Section IV. Simulated verifications will be presented in Section V, and concluding remarks will be given in Section VI.

Notation: Subscripts "f", "md", and "e" refer to false alarm, missed detection, and average error respectively; subscripts "s" and "h" refer to fusion with soft information and fusion with hard information, respectively. $x \sim \mathcal{CN}(\mu, \sigma^2)$ denotes a complex Gaussian random variable x with mean μ and variance σ^2; $b \sim \text{Bernoulli}(p_0, 1-p_0)$ denotes a Bernoulli random variable b with probability of 0 to be p_0; $u \sim \mathcal{U}(a,b)$ denotes a real random variable u uniformly distributed over interval $[a,b]$. $f(\gamma) \sim g(\gamma)$ denotes two functions of γ with $\lim_{\gamma\to+\infty}(f(\gamma)/g(\gamma)) = k$, where k is a nonzero constant.

II. Problem Formulation

In cognitive radio networks, the secondary users need to sense the spectrum usage by the primary users. The performance of spectrum sensing depends heavily on the signal strength at the secondary users. However, the signal strength at a single secondary user can be very low due to channel fading. Thus, cooperation among secondary users can be utilized to improve the sensing performance, as suggested in [3]–[5], [8], and [12]. In this section, we will introduce the signal model at the spectrum sensing users, their cooperation strategies and the performance metric of spectrum sensing in terms of the diversity order.

A. The Signal at Sensing Users

In the spectrum sensing process, the sensing users observe signals under the following two hypotheses:

$$\begin{aligned} H_0 &: \text{absence of primary user at the spectrum band of interest} \\ H_1 &: \text{presence of primary user at the spectrum band of interest.} \end{aligned}$$

We adopt the signal model in [5], where the channels between the primary and the sensing users are Rayleigh fading with additive white Gaussian noise (AWGN). Then the received signal at the sensing user is given by [5]

$$\begin{aligned} & r|H_0 = n \sim \mathcal{CN}\left(0, \sigma_n^2\right) \\ \text{or} \quad & r|H_1 = hx + n \sim \mathcal{CN}\left(0, E_x\sigma_{\text{h}}^2 + \sigma_n^2\right) \end{aligned}$$

where n is the AWGN with variance σ_n^2, h is the channel coefficient with variance σ_{h}^2 and x is the signal from the primary user with energy E_x. Suppose the sensing users have the information of the noise variance, hence, without loss of generality, we normalize the noise variance to 1. Accordingly, the signal at the sensing users becomes

$$\begin{aligned} & r|H_0 = n \sim \mathcal{CN}(0,1) \\ \text{or} \quad & r|H_1 = hx + n \sim \mathcal{CN}(0, \gamma+1) \end{aligned} \tag{1}$$

where $\gamma \triangleq E_x\sigma_{\text{h}}^2/\sigma_n^2$ is the average signal-to-noise ratio (SNR) at the sensing users.

With geographically distributed sensing users, it is reasonable to assume that they experience independent fading channels. Thus, the received signals for different sensing users r_is are conditionally independent under each hypothesis.

B. Cooperative Strategies

Cooperative spectrum sensing requires the cooperation among multiple sensing users. In our analysis, a fusion center collects information from all secondary users and facilitates their cooperation. Ideally, the cooperation benefit is maximized if all sensing information from all secondary users reaches the fusion center without any loss. This condition, however, can not always be satisfied due to the limited spectrum resource available to the secondary user system. Hence, we will next consider two types of sensing strategies, namely cooperative multiuser sensing with soft information fusion and cooperative multiuser sensing with hard information fusion.

1) Multiuser Sensing With Soft Information Fusion: In this case, the fusion center can obtain the information from the distributed secondary users perfectly. This provides a best case scenario for cooperative sensing among multiple secondary users. Although this may not be practically achievable, it does provide a useful bound on the multiuser sensing performance. Moreover, this is also a good model for the case where multiple independent faded copies of the primary user's signal are collected at a single secondary user. For example, multiple receive antennas with appropriate antenna spacing can provide independent faded copies of the signal, or in the case of fast fading scenario, the signals from different time slots are independently distributed. In these cases, the single secondary user can be regarded as the fusion center and the different sources of independent faded signals can be regarded as the multiple spectrum sensing nodes with lossless transmission to the fusion center, leading to soft information fusion.

2) Multiuser Sensing With Hard Information Fusion: In a more practical multiuser setup, each distributed secondary user senses the spectrum usage and then only transmits the one bit sensing decision, "0" for absence of primary users or "1" for presence of primary users, to the fusion center.

C. Performance Metric and Diversity Order

In traditional signal detection problems, the receiver operating characteristic (ROC) curves (false alarm probability $P_{\rm f}$ versus missed detection probability $P_{\rm md}$) are generally used to graphically illustrate the detection performance [7]. Every ROC curve is plotted under a certain combination of the system parameters such as SNR, number of cooperative users and so on. As a result, they do not provide an explicit quantitative relationship between the system parameters and the system metrics (false alarm, missed detection, and average detection error probabilities) [3]. Hence, to better illustrate the effects of the system parameters on the performance, in this paper, we analyze each single system metric as a function of the system parameter variables.

As introduced in Section I, the performance of spectrum sensing is indicated by the false alarm probability $P_{\rm f}$, the missed detection probability $P_{\rm md}$ and the average error probability $P_{\rm e}$. $P_{\rm f}$ is the probability of deciding on H_1 when H_0 is true (type I error); $P_{\rm md}$ is the probability of deciding on H_0 (type II error) when H_1 is true; $P_{\rm e}$ is the average probability of making a wrong decision. Physically, $P_{\rm f}$ determines the capability of detecting the available spectrum resource, and thus the *efficiency* of the system; and $P_{\rm md}$ indicates the level of interference that the secondary user system introduces to the primary user system, and thus the *reliability* of the system. As a result, $P_{\rm e}$ combines the efficiency and reliability considerations. We denote the probability of the absence of the primary user (H_0) as α and thus that of the presence of the primary user (H_1) as $(1-\alpha)$, then the average error probability is $P_{\rm e}=\alpha P_{\rm f}+(1-\alpha)P_{\rm md}$.

The concept of diversity was introduced in wireless communications to quantify the effects of independent fading in space, time, frequency or code space on the improvement of the system performance [11]. Quantitatively, the diversity order is defined as

$$d=-\lim_{{\rm SNR}\to+\infty}\frac{\log P}{\log {\rm SNR}}$$

where P can be the bit error rate or the outage probability of the communication system and SNR is the average signal-to-noise ratio. In cooperative sensing, the fusion center also receives multiple copies of the original signal under independent fading. Hence, the sensing performance is expected to exhibit a similar behavior. Here we define the diversity order in sensing scenarios as

$$d_*=-\lim_{\gamma\to+\infty}\frac{\log P_*}{\log\gamma}$$

where $*$ can be f (false alarm), md (missed detection) or e (average error). Accordingly, there will be false alarm diversity $d_{\rm f}$, missed detection diversity $d_{\rm md}$ and average error diversity $d_{\rm e}$. Obviously, $d_{\rm e}=\min\{d_{\rm f},d_{\rm md}\}$ when $d_{\rm f}\neq d_{\rm md}$. In the following sections, we will quantify the diversity order of spectrum sensing according to the definitions above for three cases: single-user sensing, multiuser cooperative sensing with soft information fusion and multiuser cooperative sensing with hard information fusion. In addition, we will show that, though the diversity order is defined in the limit when $\gamma\to+\infty$, it actually shows up quite early at low SNR.

III. Single-User Sensing

To achieve uniformly most powerful detection performance, we use the Neyman–Pearson (NP) detector [7]. With our signal model, the NP test is the likelihood ratio test:

$$\lambda=|r|^2\gtrless_{H_0}^{H_1}\theta \tag{2}$$

where θ is the threshold of the test. According to (1), the distribution of the decision statistic is

$$\begin{aligned}f(\lambda|H_0)&=e^{-\lambda}\quad(\lambda>0)\\ f(\lambda|H_1)&=\frac{1}{\gamma+1}e^{-\frac{\lambda}{\gamma+1}}\quad(\lambda>0).\end{aligned} \tag{3}$$

Hence, the probabilities of false alarm and missed detection are, respectively,

$$P_{\rm f}=\int_\theta^{+\infty}f(\lambda|H_0)d\lambda=\int_\theta^{+\infty}e^{-\lambda}d\lambda=e^{-\theta} \tag{4}$$

and

$$P_{\rm md}=\int_0^\theta f(\lambda|H_1)d\lambda=\int_0^\theta\frac{1}{\gamma+1}e^{-\frac{\lambda}{\gamma+1}}d\lambda=1-e^{-\frac{\theta}{\gamma+1}}. \tag{5}$$

A. Diversity Order When Minimizing $P_{\rm e}$

As shown in (4) and (5), the performance metrics $P_{\rm f}$, $P_{\rm md}$ and $P_{\rm e}$ all rely on the choice of the decision threshold θ. Clearly, one may choose different thresholds when optimizing different performance metrics. Recall that $P_{\rm f}$ captures the cognitive system's *efficiency* while $P_{\rm md}$ captures its *reliability*. To balance the system efficiency and reliability, we will optimize the threshold θ by minimizing the average error probability $P_{\rm e}=\alpha P_{\rm f}+(1-\alpha)P_{\rm md}$. Setting $dP_{\rm e}/d\theta=0$ and solving for θ, we obtain the optimum threshold as

$$\theta^o=\left(1+\frac{1}{\gamma}\right)\log\left[\frac{\alpha}{1-\alpha}(\gamma+1)\right] \tag{6}$$

where $\log$ is base-e throughout this paper unless otherwise specified. Using this threshold, as $\gamma\to+\infty$, we have

$$\begin{aligned}P_{\rm f}&=e^{-\theta}=e^{-\left(1+\frac{1}{\gamma}\right)\log\left[\frac{\alpha}{1-\alpha}(\gamma+1)\right]}\\ &=(\gamma+1)^{-\left(1+\frac{1}{\gamma}\right)}\left(\frac{\alpha}{1-\alpha}\right)^{-\left(1+\frac{1}{\gamma}\right)}\sim(1+\gamma)^{-1}\end{aligned} \tag{7}$$

and

$$P_{\mathrm{md}} = 1 - e^{-\frac{\theta}{\gamma+1}} = 1 - e^{-\frac{1}{\gamma}\log\left[\frac{\alpha}{1-\alpha}(\gamma+1)\right]} \sim \gamma^{-1}\log\left[\frac{\alpha}{1-\alpha}(\gamma+1)\right]. \quad (8)$$

Thus, their respective diversity orders can be obtained as

$$\begin{aligned} d_{\mathrm{f}} &= -\lim_{\gamma\to+\infty}\frac{\log P_{\mathrm{f}}}{\log\gamma} = 1 \\ d_{\mathrm{md}} &= -\lim_{\gamma\to+\infty}\frac{\log P_{\mathrm{md}}}{\log\gamma} = 1 \\ d_{\mathrm{e}} &= \min(d_{\mathrm{f}}, d_{\mathrm{md}}) = 1. \end{aligned} \quad (9)$$

Accordingly, we establish the following result.

Theorem 1: For single-user spectrum sensing, when the threshold θ is chosen to minimize the average error probability P_{e} as in (6), the diversity order of the NP detector is $d_{\mathrm{e}} = d_{\mathrm{f}} = d_{\mathrm{md}} = 1$.

This theorem is quite intuitive since any single sensing user only has one copy of the original signal going through the fading channel and it is well known that the probability of deep fading in this case is proportional to γ^{-1} (see, e.g., [11]).

From the analysis above, we see that the *a priori* probabilities of the hypotheses α and $(1-\alpha)$ do not affect the diversity orders of the performance. Without loss of generality, to simplify the following analyses on the diversity orders, we choose $\alpha = 1/2$ for the rest of this paper.

B. False Alarm Diversity Versus Missed Detection SNR Gain

In Theorem 1, we choose θ^o to minimize the average detection error probability P_{e}, which emphasizes equally on the system's efficiency and reliability. However, in some systems, the two features may have different levels of importance. Thus, we will next analyze P_{f} and P_{md} separately. Our analysis will reveal an interesting tradeoff between the system efficiency and reliability. This tradeoff can be exploited to achieve the desirable P_{f} and P_{md} performance and accordingly the preferable spectrum usage efficiency and interference level.

From (4) and (6), we notice that, if one changes the threshold to $\theta' = d_0\theta^o$, then the false alarm diversity order changes from 1 in (9) to

$$d'_{\mathrm{f}} = -\lim_{\gamma\to+\infty}\frac{\log P'_{\mathrm{f}}}{\log\gamma} = -\lim_{\gamma\to+\infty}\frac{-d_0\left(1+\frac{1}{\gamma}\right)\log(\gamma+1)}{\log\gamma} = d_0. \quad (10)$$

On the other hand, with this new threshold θ', as $\gamma \to +\infty$, we have

$$P'_{\mathrm{md}} = 1 - e^{-\frac{\theta'}{\gamma+1}} = 1 - e^{-\frac{d_0}{\gamma}\log(\gamma+1)} \sim d_0\gamma^{-1}\log(\gamma+1). \quad (11)$$

This implies that $d'_{\mathrm{md}} = 1$, which is identical to d_{md} in (9) with θ^o. In other words, the missed detection diversity order remains unaltered. However, the scaler difference between (8) and (11) suggests that, to ensure $P'_{\mathrm{md}} \approx P_{\mathrm{md}}$ as $\gamma \to +\infty$, ones needs $\gamma' = d_0\gamma$. This implies that the missed detection probability P_{md} exhibits a $-10\log_{10} d_0$ dB SNR gain (or equivalently $10\log_{10} d_0$ dB SNR loss) when the threshold is chosen as $\theta = d_0\theta^o$. This interesting phenomenon may result from the fact that the false alarm probability is the right tail of the Rayleigh distribution which decays very rapidly; whereas the missed detection is the left tail of the Rayleigh distribution which decays quite slowly.

We summarize the tradeoff between the false alarm diversity and the missed detection SNR gain in the following corollary.

Corollary 1: For single-user spectrum sensing, when the threshold is set to $d_0\theta^o$ with θ^o given in (6), the false alarm diversity order becomes $d_{\mathrm{f}} = d_0$, while the missed detection diversity order remains $d_{\mathrm{md}} = 1$ but the P_{md} curve exhibits a $-\log_{10} d_0$ dB SNR gain.

The tradeoff between the false alarm diversity and the missed detection SNR gain presented above provides system designers with a flexible tool to achieve the desirable tradeoff between the spectrum usage efficiency of the secondary users and the reliability of the primary users. For example, if the primary users in the cognitive system are capable of interference suppression and the spectrum usage efficiency is of major concern for the system designer, then, by the properties above, the secondary users can set the threshold as $d_0\theta^o$ with $d_0 > 1$. By this means, the false alarm diversity is increased to $d_0(> 1)$ by sacrificing a $10\log_{10} d_0$ dB SNR loss for the missed detection probability. On the other hand, if the primary users are vulnerable to interference and the performance of primary users in the cognitive system is of the major concern, then, the secondary users can set the threshold as $d_0\theta^o$ with $0 < d_0 < 1$. As a result, there will be a $-10\log_{10} d_0$ dB SNR gain for the missed detection probability by reducing the false alarm diversity to $d_0(< 1)$.

As shown later in Section IV-B, this flexibility of the false alarm diversity can also be utilized to maximize the diversity order of the multiuser sensing with hard information fusion.

IV. Multiuser Sensing

A. Soft Information Fusion

With soft information fusion strategy, the fusion center receives $r_1, r_2, \ldots, r_N$ from the distributed sensing users, where N is the total number of cooperative sensing users and r_is are conditionally independent identically distributed (i.i.d.) under both H_0 and H_1. Similar to Section III, the NP test is

$$\lambda_{\mathrm{s}} = \sum_{i=1}^{N}|r_i|^2 \gtrless_{H_0}^{H_1} \theta_{\mathrm{s}} \quad (12)$$

where the subscript "s" refers to soft information fusion.

Since r_i's are conditionally independent, and according to (1), we have

$$\begin{aligned} f(\lambda_{\mathrm{s}}|H_0) &= \lambda_{\mathrm{s}}^{N-1}\frac{e^{-\lambda_{\mathrm{s}}}}{(N-1)!} \quad (\lambda_{\mathrm{s}} > 0) \\ f(\lambda_{\mathrm{s}}|H_1) &= \lambda_{\mathrm{s}}^{N-1}\frac{e^{-\frac{\lambda_{\mathrm{s}}}{\gamma+1}}}{(N-1)!(\gamma+1)^N} \quad (\lambda_{\mathrm{s}} > 0). \end{aligned} \quad (13)$$

Hence, the probabilities of false alarm and missed detection are, respectively,

$$P_{\mathrm{f,s}} = \int_{\theta_{\mathrm{s}}}^{+\infty} f(\lambda_{\mathrm{s}}|H_0)d\lambda_{\mathrm{s}} = \left(\sum_{i=0}^{N-1}\frac{\theta_{\mathrm{s}}^i}{i!}\right)e^{-\theta_{\mathrm{s}}} \quad (14)$$

and

$$P_{\mathrm{md,s}} = \int_0^{\theta_{\mathrm{s}}} f(\lambda_{\mathrm{s}}|H_1)d\lambda_{\mathrm{s}} = \left(\sum_{i=N}^{+\infty} \frac{\theta_{\mathrm{s}}^i}{i!(\gamma+1)^i}\right) e^{-\frac{\theta_{\mathrm{s}}}{\gamma+1}}. \quad (15)$$

Accordingly, the average error probability is $P_{\mathrm{e,s}} = (1/2)P_{\mathrm{f,s}} + (1/2)P_{\mathrm{md,s}}$. Similar to Section III, minimizing $P_{\mathrm{e},s}$ by taking $dP_{\mathrm{e},s}/d\theta_{\mathrm{s}} = 0$, we obtain the optimum threshold as

$$\theta_{\mathrm{s}}^o = N\left(1+\frac{1}{\gamma}\right)\log(\gamma+1). \quad (16)$$

Using this threshold, we establish the following theorem.

Theorem 2: For multiuser sensing with soft information fusion, when the threshold θ_{s} is chosen as in (16) to minimize the average error probability $P_{\mathrm{e,s}}$, the diversity order of the NP detector is $d_{\mathrm{e,s}} = d_{\mathrm{f,s}} = d_{\mathrm{md,s}} = N$, where N is the number of cooperative users.

Proof: See Appendix I. ■

This theorem is also intuitive in that the fusion center has copies of the original received signals from N independently fading channels. Similar to Section III-B, we can also choose the threshold as $\theta_{\mathrm{s}}' = d_0\theta_{\mathrm{s}}^o$, where d_0 can be any positive number and is not necessarily integer, to increase the false alarm diversity to d_0N while keeping the missed detection diversity unaltered at N. In this case, there is also a tradeoff between the missed detection SNR gain and the false alarm diversity. For the false alarm diversity to be d_0N, the missed detection probability will exhibit a $-10\log_{10} d_0$ dB SNR gain (or equivalently $10\log_{10} d_0$ dB SNR loss).

B. Hard Information Fusion

With the hard information fusion strategy, each sensing user makes its own local hard decision and then sends the binary decision b_i to the fusion center. For simplicity, we assume that all distributed sensing users employ the same threshold θ_{l} for their local decisions where subscript "l" stands for local. The corresponding local false alarm and missed detection probabilities are denoted as $P_{\mathrm{f,l}}$ and $P_{\mathrm{md,l}}$, respectively. Clearly, b_i follows conditionally i.i.d. Bernoulli distribution with $(1-P_{\mathrm{f,l}})$ and $P_{\mathrm{md,l}}$ as the probabilities of value 0 under H_0 and H_1, respectively; that is

$$\begin{aligned} b_i|H_0 &\sim \text{Bernoulli}(1-P_{\mathrm{f,l}}, P_{\mathrm{f,l}}) \\ b_i|H_1 &\sim \text{Bernoulli}(P_{\mathrm{md,l}}, 1-P_{\mathrm{md,l}}). \end{aligned} \quad (17)$$

In this case, the NP test becomes

$$\lambda_{\mathrm{h}} = \sum_{i=1}^N b_i \gtrless_{H_0}^{H_1} \theta_{\mathrm{h}}, \quad \theta_{\mathrm{h}} = 1,2,\ldots,N \quad (18)$$

where subscript "h" refers to hard information fusion. Accordingly, the distribution of λ_{h} is

$$\begin{aligned} f(\lambda_{\mathrm{h}}|H_0) &= \binom{N}{\lambda_{\mathrm{h}}} P_{\mathrm{f,l}}^{\lambda_{\mathrm{h}}}(1-P_{\mathrm{f,l}})^{N-\lambda_{\mathrm{h}}}, \\ &\quad \lambda_{\mathrm{h}} = 0,1,\ldots,N; \\ f(\lambda_{\mathrm{h}}|H_1) &= \binom{N}{\lambda_{\mathrm{h}}}(1-P_{\mathrm{md,l}})^{\lambda_{\mathrm{h}}} P_{\mathrm{md,l}}^{N-\lambda_{\mathrm{h}}}, \\ &\quad \lambda_{\mathrm{h}} = 0,1,\ldots,N. \end{aligned} \quad (19)$$

In hard information fusion, there are two levels of decision making, each level having its own decision performance. For the local decision, there are diversity orders for the local false alarm probabilities $(d_{\mathrm{f,l}})$ and local missed detection probabilities $(d_{\mathrm{md,l}})$. At the fusion center, there are also corresponding diversity orders for the overall hard-decision false alarm probability $(d_{\mathrm{f,h}})$, missed detection probability $(d_{\mathrm{md,h}})$ and average error probability $(d_{\mathrm{e,h}})$. Here we establish the relationship between the local decision diversity orders with the overall diversity orders at the fusion center in the following theorem.

Theorem 3: For multiuser sensing with 1-bit hard information fusion, and with the fusion center threshold θ_{h} ($\theta_{\mathrm{h}} = 1,2,\ldots,N$), the diversity orders of the NP detector are $d_{\mathrm{f,h}} = \theta_{\mathrm{h}}d_{\mathrm{f,l}}$, $d_{\mathrm{md,h}} = (N-\theta_{\mathrm{h}}+1)d_{\mathrm{md,l}}$ and $d_{\mathrm{e,h}} = \min\{\theta_{\mathrm{h}}d_{\mathrm{f,l}}, (N-\theta_{\mathrm{h}}+1)d_{\mathrm{md,l}}\}$, where N is the number of cooperative users.

Proof: See Appendix II. ■

Similar to the single-user sensing and cooperative sensing with soft information fusion, cooperative sensing with hard information fusion also provides the system designers with the flexibility of balancing between the spectrum efficiency of the secondary users (via $d_{\mathrm{f,h}}$) and the reliability of the primary users (via $d_{\mathrm{md,h}}$) by the choice of the threshold θ_{h}. A larger θ_{h} will improve the false alarm performance, leading to higher spectrum usage efficiency of the secondary users; while a smaller θ_{h} will improve the missed detection performance, leading to enhanced reliability of the primary users. However, it is worth noting that the tradeoff and flexibility here are very different from what we have discussed in Corollary 1 for the single-user sensing and the multiuser soft-fusion cases. In previous cases, the tradeoff was between the false alarm *diversity* and the missed detection *SNR gain*, while here in the case of multiuser hard-fusion, the tradeoff is between the false alarm *diversity* and the missed detection *diversity*.

Note that, though the local threshold θ_{l} does not appear explicitly in Theorem 3, it affects the overall system implicitly via $d_{\mathrm{f,l}}$ and $d_{\mathrm{md,l}}$. Hence, if one opts to minimize $P_{\mathrm{e,h}} = (1/2)P_{\mathrm{f,h}} + (1/2)P_{\mathrm{md,h}}$, one needs to jointly choose both the optimum local threshold θ_{l} at the individual sensing users and the optimum hard decision threshold θ_{h} at the fusion center. However, under this fusion rule, θ_{l} has a very complicated form in $P_{\mathrm{e,h}}$ through parameters $P_{\mathrm{f,l}}$ and $P_{\mathrm{md,l}}$, rendering the optimization process mathematically intractable. Even with numerical techniques, the optimization over θ_{l} still requires the information of the total number of distributed sensing users in the network N, which is not always available to the secondary users in real applications. However, with Theorem 3, one can optimize the overall hard decision fusion performance from the diversity perspective with different strategies as detailed in the following two scenarios.

1) Number of Cooperative Users N Unknown: In this case, the sensing users can only perform their optimum detection locally. From Theorem 1, the local threshold is $\theta_{\mathrm{l}} = \theta^o$ and the local diversities are $d_{\mathrm{f,l}} = d_{\mathrm{md,l}} = 1$. According to Theorem 3, the diversity orders at the fusion center are $d_{\mathrm{f,h}} = \theta_{\mathrm{h}}$, $d_{\mathrm{md,h}} = N - \theta_{\mathrm{h}} + 1$ and $d_{\mathrm{e,h}} = \min\{\theta_{\mathrm{h}}, N-\theta_{\mathrm{h}}+1\}$.

With equal emphasis on false alarm and missed detection performance, by maximizing $\min\{\theta_{\rm h}, N-\theta_{\rm h}+1\}$, we obtain the optimum threshold $\theta_{\rm h}^o$ at the fusion center that maximizes the detection error diversity.

Corollary 2: For multiuser sensing with 1-bit hard information fusion, and with each sensing user using locally optimum threshold $\theta_{\rm l} = \theta^o$, the optimum threshold at the fusion center in the sense of maximizing the detection error diversity is $\theta_{\rm h}^o = \lfloor (N+1)/2 \rfloor$ or $\theta_{\rm h}^o = \lceil (N+1)/2 \rceil$ with $d_{\rm e,h} = \lfloor (N+1)/2 \rfloor$, where N is the number of cooperative users.

In this case, the strategy is the so-termed majority-fusion rule. Notice that under this rule, about half of the diversity is lost compared with the soft information fusion. This indicates that the hard decision at each local sensing user leads to considerable information loss of the received signal.

2) Number of Cooperative Users N Known: From Corollary 1, we have seen that the false alarm diversity at each local sensing user is flexible. It is shown in the following corollary that this property can be utilized to maximize the average error diversity.

Corollary 3: For multiuser sensing with 1-bit hard information fusion, if each distributed sensing user knows the number of cooperative users N, the average error diversity order at the fusion center can be maximized by choosing local decision threshold $\theta_{\rm l} = N\theta^o$ with θ^o given in (6) and the fusion decision threshold $\theta_{\rm h} = 1$.

Proof: See Appendix III. ■

Notice that the decision strategy turns out to be the so-termed OR-fusion rule. From this corollary, we see that with the information of total cooperative user number available at each local sensing user, the diversity performance of the average detection error probability of the hard information fusion equals that of the soft information fusion ($d_{\rm e,h} = d_{\rm e,s} = N$). In other words, the information of cooperative user number N completely compensates for the loss of information by local hard decisions, in terms of the detection error diversity. However, we should also notice that though the diversity performance of the two cases are identical, their average error probability performance are still different. As detailed in Section III-B, there is a $10\log N$ SNR loss for the missed detection probability $P_{\rm md,l}$ by setting $\theta_{\rm l} = N\theta^o$ (such that $d_{\rm f,l} = N$).

Remarks: As stated in Section II-B, our results on multiuser sensing with soft information fusion can be readily applied to the case of combining signals from different multiple time slots at a single user, as long as the combined received signals experience independent fading. In this case, if the system also uses cooperative multiuser sensing with hard information fusion among these multiple time slot sensing users, our analysis in this section can be readily extended by combining the results in Theorems 2 and 3. In addition, for the correlated fading case, intuitively, we expect that the diversity orders equal the rank of the correlation matrix of the received signals, which will be justified in the following section.

V. SIMULATIONS

A. Single-User Sensing

For single-user sensing, we use the test given by (2) and the threshold by (6) to obtain the average error probability, false alarm probability and the missed detection probability. These probabilities are shown as the solid curves in Fig. 1. All three curves exhibit the same slope, indicating diversity orders $d_{\rm f} = d_{\rm md} = d_{\rm e} = 1$.[1]

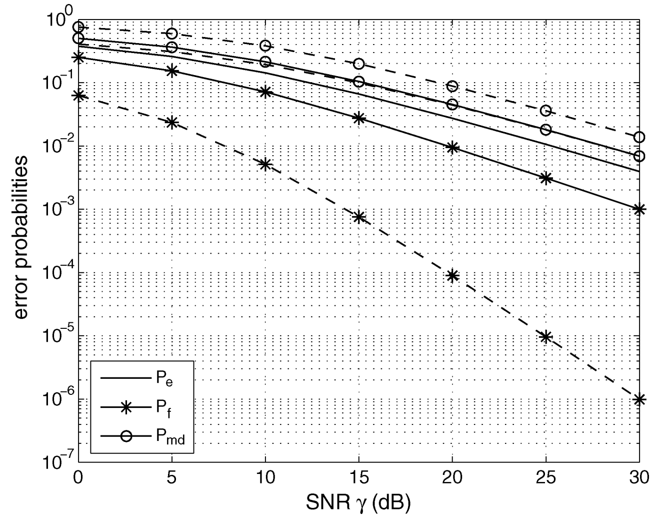


Fig. 1. Single-user sensing. Solid curves: threshold $\theta = \theta^o$; Dashed curves: threshold $\theta = 2\theta^o$.

To illustrate the tradeoff between the false alarm diversity and missed detection SNR gain (loss) discussed in Section III-B, we change the threshold to $\theta' = 2\theta^o$ and obtain the dashed curves in Fig. 1. From the figure, we see that $d'_{\rm f} = 2$ and $d'_{\rm md} = d_{\rm md} = 1$, as predicted Corollary 1. In addition, by comparing the solid curve with the dashed one for $P_{\rm md}$, one can easily verify that at high SNR, there is an approximately $10\log_{10} d_0 = 3$ dB SNR loss. This is the price paid for the increase of the diversity for the false alarm probability.

B. Multiuser Sensing With Soft Information Fusion

Here, we simulate the multiuser sensing with soft information fusion strategy. The total number of cooperating users is $N = 5$. From the analysis in Section IV-A, we expect the relative performance of $P_{\rm e,s}$, $P_{\rm f,s}$ and $P_{\rm md,s}$ to be similar to the single-user sensing case, except for the diversity order of 5. This is verified in Fig. 2. The solid curves show that with the threshold minimizing the average error probability, the diversity orders are $d_{\rm e,s} = d_{f,s} = d_{md,s} = N = 5$. In multiuser sensing with soft information fusion, we can also set the threshold to $\theta'_{\rm s} = d_0\theta_{\rm s}^o$ to make $d_{f,s} = d_0 N$. The dashed curves in Fig. 2 show $P_{\rm f,s}$ and $P_{\rm md,s}$ with $d_0 = 0.5$. Similar to the single-user sensing case, comparing the solid curve with the dashed one for $P_{\rm md,s}$, we verify that at high SNR, there is approximately $-10\log_{10} d_0 = 3$ dB SNR gain for the decrease of the false alarm diversity (from 5 to 2.5).

In the simulations above, the fading coefficients at each user are assumed to be independent. However, for multitime-slot sensing which can be modeled as multiuser sensing with soft information fusion, the fading coefficients can be correlated. To investigate the diversities in this case, we plot the simulation results of the detector under the threshold $\eta_{\rm s}^o$ in Fig. 3. Notice

[1]For all the simulations performed in this section, the iteration ends when at least 10 errors have occurred or 10^8 iterations are completed.

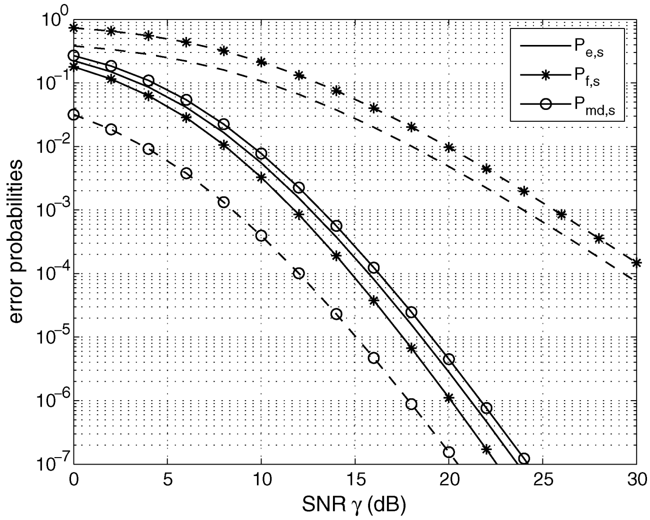


Fig. 2. $N = 5$ multiuser sensing with soft decision fusion. Solid curves: threshold $\theta = \theta_s^o$; dashed curves: threshold $\theta = 0.5\theta_s^o$.

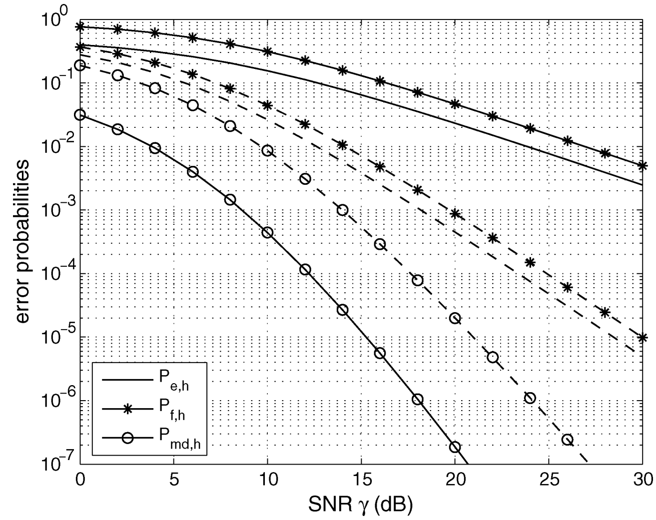


Fig. 4. $N = 5$ multiuser sensing with hard decision fusion. Solid curves: threshold $\theta_l = \theta^o$, $\theta_h = 1$; dashed curves: threshold $\theta_l = \theta^o$, $\theta_h = 2$.

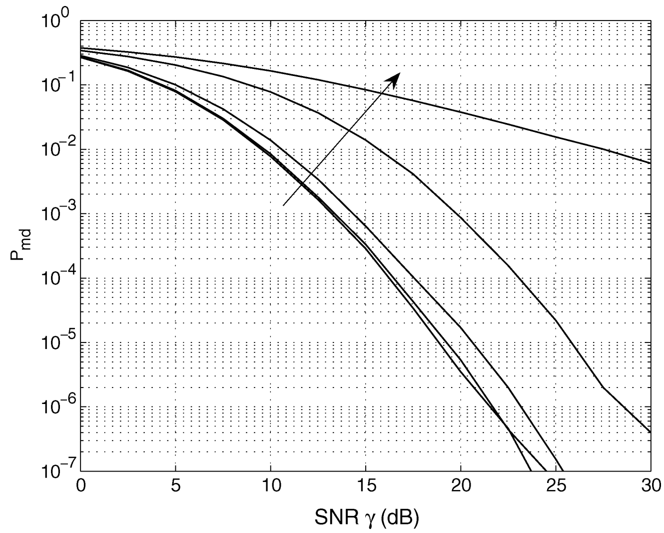


Fig. 3. $N = 5$ multiuser sensing with soft decision fusion on correlated signals with correlation $\mathbb{E}[h_i^* h_j] = r^{|i-j|}$. In the direction of arrow: $r = 0$, $r = 0.2$, $r = 0.5$, $r = 0.9$, $r = 1$, respectively.

that in this case, the signals under hypothesis H_0 remains unaltered, thus it suffices to give the missed detection performances only. In this simulation, the number of time slots $N = 5$ and the correlations of the fading coefficients are assumed to be $\mathbb{E}[h_i^* h_j] = r^{|i-j|}$ where i and j are indexes for the time slot. Under this assumption, the correlation matrix is

$$\Lambda_{\mathbf{h}} = \mathbb{E}[\mathbf{h}^H \mathbf{h}] = \begin{bmatrix} 1 & r & r^2 & r^3 & r^4 \\ r & 1 & r & r^2 & r^3 \\ r^2 & r & 1 & r & r^2 \\ r^3 & r^2 & r & 1 & r \\ r^4 & r^3 & r^2 & r & 1 \end{bmatrix}.$$

In Fig. 3, we see that when the correlation (r) among the fading coefficients increases, the performance degrades. However, as long as the correlation matrix $\Lambda_{\mathbf{h}}$ is full-ranked, the diversity results obtained under the independent fading scenario still hold. Only when the matrix loses rank $(r = 1)$, the diversity reduces to 1.

C. Multiuser Sensing With Hard Information Fusion

For the multiuser sensing with hard information fusion in Section IV-B, we first consider the case where the number of cooperative users N is not available to the individual sensing users. Then, each distributed sensing user makes the locally optimum hard decision, i.e., $\theta_l = \theta^o$ as defined in (6). In this case, as Theorem 3 dictates, the false alarm diversity, the missed detection diversity and the average error diversity are all heavily dependent on the threshold θ_h at the fusion center. Fig. 4 shows the behavior of $P_{f,h}$, $P_{md,h}$ and $P_{e,h}$ of the hard information fusion strategy with $\theta_h = 1$ (solid curves) and $\theta_h = 2$ (dashed curves) when the number of cooperative users is $N = 5$. When $\theta_h = 1$, the solid curves show that $d_{f,h} = 1$, $d_{md,h} = 5$ and $d_{e,h} = 1$; when $\theta_h = 2$, the dashed curves show that $d_{f,h} = 2$, $d_{md,h} = 4$ and $d_{e,h} = 2$. These results are consistent with Theorem 3 and illustrate the tradeoff between the false alarm diversity and the missed detection diversity.

With locally optimum hard decision $(\theta_l = \theta^o)$ at distributed sensing users, the threshold maximizing the average error diversity should be chosen as $\lfloor (N+1)/2 \rfloor$ or $\lceil (N+1)/2 \rceil$ according to Corollary 2. This result is shown in Fig. 5 when $N = 5$. From this figure, we see that $\theta_h = 3$ gives the maximum average error diversity. Compared with the error performance under soft information fusion, we see here that this locally optimum hard decision strategy suffers from a large loss of diversity.

When the distributed sensing users know the number of cooperative users in the network, then the flexibility of the false alarm diversity in Corollary 1 can be utilized to maximize the diversities of the hard information fusion. In this case, the local decision threshold is $\theta_l = N\theta^o$, where θ^o is defined in (6). Fig. 6 shows the performance with this strategy and compares this with that of the soft information fusion strategy. From this figure, we see that in terms of diversity order d_e, the hard information fusion with adjusted local threshold equals the soft information fusion. However, the soft information fusion has a huge SNR advantage over the hard information fusion on the missed detection and average error probabilities. This is due to the fact that in the local decisions, a $10\log_{10} 5 = 7$ dB SNR loss of

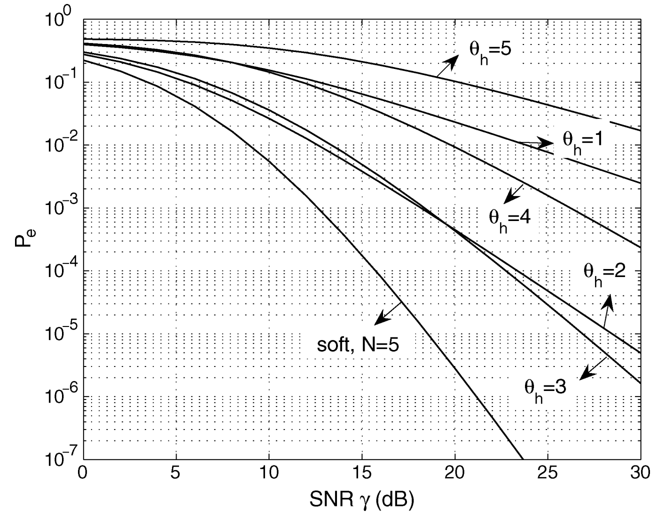


Fig. 5. $N = 5$ multiuser sensing with soft information fusion and hard information fusion with $\theta_l = \theta^o$ and various θ_hs.

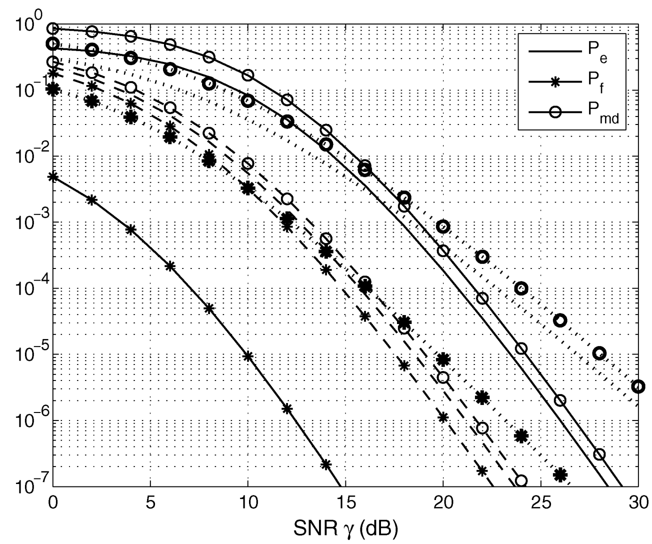


Fig. 6. $N = 5$ multiuser sensing with OR-fusion rule with and $\theta_l = N\theta^o$ (solid curves), majority-fusion rule with $\theta_l = \theta^o$ (dotted curves) and soft information fusion (dashed curves).

missed detection probability is introduced to increase the false alarm diversity from $d_{f,l} = 1$ to $d_{f,l} = 5$ by setting $\theta_l = 5\theta^o$. This also explains why the false alarm performance in this hard information fusion strategy with knowledge of cooperative user number is better than that in the soft information fusion strategy. In addition, compared with the dotted curves, we see that this strategy recovers the diversity loss introduced by the locally optimum hard decision.

D. Low SNR Performance Comparisons

The analysis in this paper focuses on the diversity order which is meaningful only at high SNR. However, from the simulation results above, we see that the diversity shows up quite early in SNR. Actually, most cases with better diversity order also results in better performance as shown at low SNR in Figs. 1–6.

To further illustrate the relative performance of various decision strategies in low SNR range, we show in Fig. 7 the average detection error of soft information fusion ($d_{e,s} = N$) with $\theta_s = \theta_s^o$, hard information fusion with $\theta_l = \theta^o$ and $\theta_h^o = \lfloor (N+1)/2 \rfloor$ as in Corollary 2 ($d_{e,h} = \lfloor (N+1)/2 \rfloor$) and hard information fusion with $\theta_l = N\theta^o$ and $\theta_h = 1$ as in Corollary 3 ($d_{e,h} = N$) at $\gamma = 0$ dB. Comparing the solid curve with the other two curves, we see that at low SNR, the soft information fusion is still better than the hard information fusion. This is consistent with our expectation in that the soft information fusion center collects all the information at individual sensing users and the threshold maximizing the diversity in (16) is actually optimum for any SNR value γ. Also, the performance of soft information fusion improves as the number of cooperative sensing users N increases. This means that a bigger number of cooperative sensing users not only increases the diversity order at high SNR but also boosts the performance at low SNR.

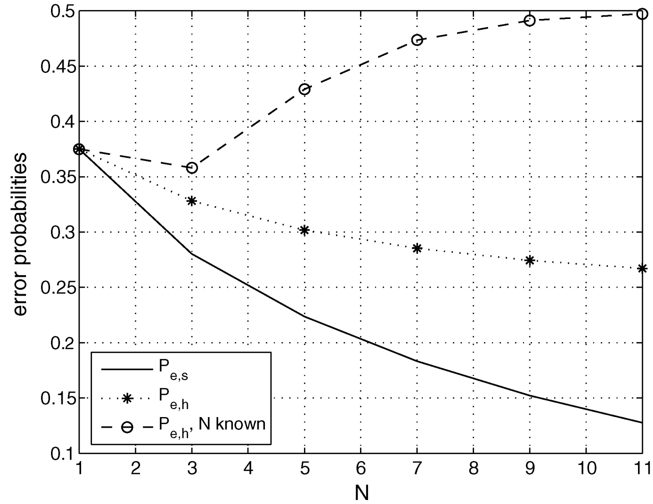


Fig. 7. Low SNR performance of different decision strategies ($\gamma = 0$ dB). Solid: soft information fusion; dotted: majority fusion with $\theta_l = \theta^o$; dashed: OR fusion with $\theta_l = N\theta^o$.

Comparing the dashed curve with the dotted one, we see that, though with the knowledge of the number of cooperative sensing users, the diversity of the hard information fusion can be maximized as dictated in Corollary 3, its low-SNR performance is greatly compromised. This is due to the fact that with the strategy in Corollary 3, the performance of local missed detection $P_{md,l}$ is sacrificed for the sake of false alarm diversity. At low SNR, the benefit of false alarm is insignificant while the loss of local missed detection performance takes dominance on the overall decision fusion performance. Also, it should be noticed that with the strategy in Corollary 3, larger number of cooperative users N causes more performance loss of local missed detection $P_{md,l}$, leading to more significantly deteriorated overall performance of the hard information fusion. With this comparison, we also obtain the conclusion that at low SNR, the majority-fusion introduced in Corollary 2 is preferred than the OR-fusion introduced in Corollary 3 for hard information fusion. This is quite intuitive since the majority-fusion rule is more robust to individual errors.

E. Simulation With Imperfect SNR Estimate

In all our strategies, one needs the SNR information to determine the decision thresholds θ^o, θ_s^o and θ_h. However, in real

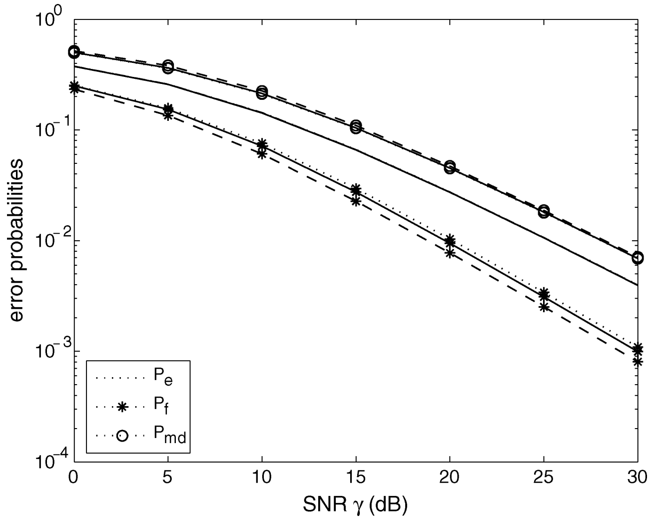


Fig. 8. Performances of single-user spectrum sensing with imperfect SNR estimate $\hat{\gamma} = u\gamma$. Solid curves: $u = 1$ (perfect); dotted curves: $u \sim \mathcal{U}(0.5, 1.5)$ (unbiased); dashed curves: $u \sim \mathcal{U}(1, 1.5)$ (biased).

applications, the SNR information can only be obtained via estimation and is thus never perfectly known. Next, we simulate our algorithms with imperfectly estimated SNR values.

First, suppose our estimated SNR $\hat{\gamma} = u\gamma$ where $u \sim \mathcal{U}(a, b)$ is the multiplicative noise factor. As discussed before, the performance of the single-user sensing is fundamental to that of the multiuser sensing case. Therefore, here we only present the effect of imperfect SNR estimate on single-user sensing in Fig. 8.

This figure shows the performances of the single-user sensing with perfect (solid), unbiased (dotted), and biased (dashed) SNR estimate. From this figure we see that there is very little difference among the performances with different SNR estimation quality. This means that our spectrum sensing algorithm is robust against imperfect SNR estimate. This is because though the threshold expression involves the SNR value, the algorithm only requires the threshold to increase or decrease in consistency with SNR.

It is also worth noting that with the biased SNR estimate, if the bias is positive as in the dashed curve case, the performance of false alarm (P_{f}) will actually get improved because we are choosing a larger threshold. We expect the opposite when the bias is negative.

VI. Conclusion

In this paper, we analyzed various cooperative spectrum sensing strategies under different scenarios. By considering both false alarm and missed detection probabilities individually and jointly via the average error probability, we found several tradeoffs between the system efficiency and reliability under three different spectrum sensing strategies. For single-user sensing and multiuser sensing with soft information fusion, the tradeoff is between the false alarm diversity gain and the missed detection SNR loss by altering the detection threshold. For multiuser sensing with hard information fusion without information of cooperative user number, there is a tradeoff between the diversities of false alarm and missed detection. In addition, under hard information fusion, with the knowledge of cooperative user number, the soft decision diversity can be achieved at a given missed detection SNR loss. With these diversity and tradeoff results, we derived the optimum threshold in each cooperative strategy to guide practical system design. Simulations have also been presented to further illustrate the analytical results and compare the various cooperative strategies.

Appendix I
Proof of Theorem 2

With $\theta_{\mathrm{s}} = N(1 + (1/\gamma))\log(\gamma + 1)$, as $\gamma \to +\infty$, $e^{-\theta_{\mathrm{s}}} = (1+\gamma)^{-N(1+(1/\gamma))} \sim (\gamma+1)^{-N}$ and $\sum_{i=0}^{N-1} \theta_{\mathrm{s}}^i / i! = \sum_{i=0}^{N-1} N^i (1 + (1/\gamma))^i (\log(\gamma + 1))^i / i! \sim N^{N-1}(\log(\gamma + 1))^{N-1}/(N - 1)!$. Thus, according to (14), $P_{f,s} \sim (N^{N-1}/(N-1)!)(\log(\gamma+1))^{N-1}(\gamma+1)^{-N}$ and

$$d_{f,s} = -\lim_{\gamma\to+\infty} \frac{\log P_{f,s}}{\log \gamma} = N.$$

Also, with θ_{s} defined above, as $\gamma \to +\infty$, $e^{-\theta_{\mathrm{s}}/(\gamma+1)} = e^{-(N/\gamma)\log(\gamma+1)} \to 1$ and $\sum_{i=N}^{+\infty} \theta_{\mathrm{s}}^i/(i!(\gamma + 1)^i) = \sum_{i=N}^{+\infty} N^i (1 + (1/\gamma))^i (\log(\gamma + 1))^i/(i!(\gamma + 1)^i) \sim N^N(\log(\gamma + 1))^N(\gamma + 1)^{-N}/N!$. Thus, according to (15), $P_{\mathrm{md,s}} \sim (N^N/N!)(\log(\gamma+1))^N(\gamma+1)^{-N}$ and

$$d_{\mathrm{md,s}} = -\lim_{\gamma\to+\infty} \frac{\log P_{\mathrm{md,s}}}{\log \gamma} = N.$$

Accordingly, $d_{\mathrm{e,s}} = \min(d_{\mathrm{f,s}}, d_{\mathrm{md,s}}) = N$.

Appendix II
Proof of Theorem 3

From (18) and (19)

$$P_{\mathrm{f,h}} = P(\lambda_{\mathrm{h}} \geq \theta_{\mathrm{h}} | H_0) = \sum_{i=\theta_{\mathrm{h}}}^{N} \binom{N}{i} P_{\mathrm{f,l}}^i (1 - P_{\mathrm{f,l}})^{N-i}.$$

The false alarm diversity at the local sensing decision is $d_{\mathrm{f,l}}$, so as $\gamma \to +\infty$, $P_{\mathrm{f,l}} \sim (\gamma + 1)^{-d_{\mathrm{f,l}}}$, $1 - P_{\mathrm{f,l}} \to 1$, thus $P_{\mathrm{f,l}}^i (1-P_{\mathrm{f,l}})^{N-i} \sim (\gamma+1)^{-i d_{\mathrm{f,l}}}$. In the summation of $P_{\mathrm{f,h}}$, as $\gamma \to +\infty$, the term with lowest power order of $(\gamma + 1)^{-1}$ will dominate, thus $P_{\mathrm{f,h}} \sim \binom{N}{\theta_{\mathrm{h}}}(\gamma + 1)^{-\theta_{\mathrm{h}} d_{\mathrm{f,l}}}$, and the false alarm diversity order is

$$d_{\mathrm{f,h}} = -\lim_{\gamma\to+\infty} \frac{\log P_{\mathrm{f,h}}}{\log \gamma} = \theta_{\mathrm{h}} d_{\mathrm{f,l}}, \quad \theta_{\mathrm{h}} = 1, 2, \ldots, N.$$

From (18) and (19)

$$P_{\mathrm{md,h}} = P(\lambda_{\mathrm{h}} < \theta_{\mathrm{h}} | H_1) = \sum_{i=0}^{\theta_{\mathrm{h}}-1} \binom{N}{i} (1 - P_{\mathrm{md,l}})^i P_{\mathrm{md,l}}^{N-i}.$$

From the same argument, the missed detection diversity order is

$$\begin{aligned} d_{\mathrm{md,h}} &= -\lim_{\gamma\to+\infty} \frac{\log P_{\mathrm{md,h}}}{\log \gamma} \\ &= (N - \theta_{\mathrm{h}} + 1) d_{\mathrm{md,l}}, \quad \theta_{\mathrm{h}} = 1, 2, \ldots, N. \end{aligned}$$

Appendix III
Proof of Corollary 3

From Corollary 1, we know that the local threshold can be chosen as $\theta_{\mathrm{l}} = d_0 \theta^o$ $(d_0 > 0)$. In this case, $d_{\mathrm{f,l}} = d_0$ and $d_{\mathrm{md,l}} = 1$. By Theorem 3, if the hard decision threshold is

θ_h ($\theta_h \in \{1, 2, \ldots, N\}$), the average error diversity is $d_{e,h} = \min(\theta_h d_0, N - \theta_h + 1)$. To maximize $d_{e,h}$, we need to maximize both $\theta_h d_0$ and $(N - \theta_h + 1)$ simultaneously. By maximizing the latter, we obtain $\theta_h = 1$. Then $d_{e,h} = \min(d_0, N)$. Thus, as long as $d_0 \geq N$, we obtain the maximum diversity $d_{e,h} = N$. However, as stated in Corollary 1, higher d_0 will cause higher SNR loss for the missed detection performance. Thus, we choose $d_0 = N$ to minimize the SNR loss for the missed detection performance while achieving the maximum average error diversity.

References

[1] D. Cabric, A. Tkachenko, and R. W. Brodersen, "Experimental study of spectrum sensing based on energy detection and network cooperation," presented at the ACM 1st Int. Workshop Technology Policy Accessing Spectrum (TAPAS), Boston, MA, Aug. 2006.

[2] S. Cui, J. Xiao, A. J. Goldsmith, Z. Luo, and H. V. Poor, "Estimation diversity and energy efficiency in distributed sensing," *IEEE Trans. Signal Process.*, vol. 55, no. 9, pp. 4683–4695, Sep. 2007.

[3] A. Ghasemi and E. S. Sousa, "Collaborative spectrum sensing for opportunistic access in fading environments," in *Proc. 1st IEEE Int. Symp. New Frontiers Dynamic Spectrum Access Networks (DySPAN)*, Baltimore, MD, Nov. 8–11, 2005, pp. 131–136.

[4] S. Haykin, "Cognitive radio: Brain-empowered wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 2, pp. 201–220, Feb. 2005.

[5] S. Hong, M. H. Vu, and V. Tarokh, "Cogntive sensing based on side information," presented at the IEEE Sarnoff Symp., Princeton, NJ, Apr. 28–30, 2008.

[6] J. Mitola, III and G. Q. Maguire, Jr., "Cognitive radio: Making software radios more personal," *IEEE Personal Commun.*, vol. 6, no. 4, pp. 13–18, Aug. 1999.

[7] H. V. Poor, *An Introduction to Signal Detection and Estimation*, 2nd ed. New York: Springer-Verlag, 1994.

[8] Z. Quan, S. Cui, and A. H. Sayed, "Optimal linear cooperation for spectrum sensing in cognitive radio networks," *IEEE J. Sel. Topics Signal Process.*, vol. 2, no. 1, pp. 28–40, Feb. 2008.

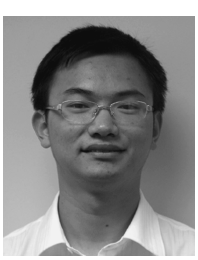
[9] G. Staple and K. Werbach, "The end of spectrum scarcity," *IEEE Spectrum*, vol. 41, no. 3, pp. 48–52, Mar. 2004.

[10] R. Tandra and A. Sahai, "SNR walls for signal detection," *IEEE J. Sel. Topics Signal Process.*, vol. 2, pp. 4–17, Feb. 2008.

[11] D. N. C. Tse and P. Viswanath, *Fundamentals of Wireless Communications*. Cambridge, U.K.: Cambridge Univ. Press, 2005.

[12] J. Unnikrishnan and V. V. Veeravalli, "Cooperative sensing for primary detection in cognitive radio," *IEEE J. Sel. Topics Signal Process.*, vol. 2, no. 1, pp. 18–27, Feb. 2008.

[13] Q. Zhao and B. M. Sadler, "A survey of dynamic spectrum access," *IEEE Signal Process. Mag.*, vol. 24, no. 3, pp. 79–89, May 2007.



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