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Designing Equally Fault-Tolerant Configurations for Kinematically Redundant Manipulators

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Abstract— **In this article, the authors examine the problem of designing nominal manipulator Jacobians that are optimally fault-tolerant to multiple joint failures. In this work, optimality is defined in terms of the worst case relative manipulability index. Building on previous work, it is shown that for a robot manipulator working in three-dimensional workspace to be equally fault-tolerant to any two simultaneous joint failures, the manipulator must have precisely six degrees of freedom. A corresponding family of Jacobians with this property is identified. It is also shown that the two-dimensional workspace problem has no such solution.**

I. INTRODUCTION

Fault-tolerant design of serial or parallel manipulators is critical for tasks requiring robots to operate in remote and hazardous environments where repair and maintenance tasks are extremely difficult [1]-[10]. In such cases, operational reliability is of prime importance. By adding kinematic redundancy to the robotic system, the robot may still be able to perform its task even if one or more joint actuators fail [11]. However, simply adding kinematic redundancy to the system does not guarantee fault tolerance [12]. One must strategically plan how the kinematic redundancy should be added to the system to ensure that fault tolerance is optimized [13].

One approach to the problem of designing fault-tolerant robots is to optimize some measure of fault tolerance. This measure can be either global, i.e., over a specified region of the workspace, or local, i.e., at a specific configuration. Global measures, such as those in [14], [15], are more appropriate for tasks that require large motions throughout the workspace, whereas local measures [11], [16] are more appropriate for dexterous operations in a relatively small location, e.g., laser pointing [6] and manipulation of nuclear material [9]. In this article we focus on a local measure called the relative manipulability index, which was first introduced in [12] to quantify the fault tolerance of kinematically redundant serial manipulators. Relative manipulability indices have also been used to study the fault tolerance of redundant Gough-Stewart platforms [17].

In the next section, we describe the relative manipulability index and its relationship to the null space of the manipulator Jacobian. In Section III, bounds are derived for the minimum relative manipulability index. These bounds motivate the goal of finding optimally fault-tolerant manipulator configurations in Section IV. Examples of optimally fault-tolerant configurations are presented. Lastly, conclusions appear in Section V.

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II. THE RELATIVE MANIPULABILITY INDEX

For a serial manipulator, the relative manipulability index is defined in terms of the manipulator Jacobian J, which relates the manipulator's joint velocity $\dot{\theta}$ to its end-effector velocity $\mathbf{v}$ by the equation

$$\mathbf{v} = J\dot{\theta}. \tag{1}$$

In this work, we will assume that the manipulator is not operating at a kinematic singularity so that J has full rank.

A joint failure significantly affects the kinematics of the robot. Two types of joint failures for serial manipulators have been examined in the literature. One type is a free-swinging failure. In this case, the failed joint becomes passive. The other type, which we will study in this work, is a locked-joint singularity. When a locked-joint failure occurs, say in joint i, that component of the joint velocity is zero. Consequently, the end-effector motion is characterized by ${}^{i}J$, i.e., the Jacobian J with its i-th column removed. Multiple locked-joint failures are handled in the same way, i.e., the corresponding columns of the Jacobian are removed.

The *relative manipulability index* corresponding to locked-joint failures in joints $i_1, \ldots, i_f$ is defined to be

$$\rho_{i_1,\cdots,i_f} = \frac{w({}^{i_1\cdots i_f}J)}{w(J)}, \tag{2}$$

where ${}^{i_1\cdots i_f}J$ denotes the manipulator Jacobian after the columns $i_1, \ldots, i_f$ corresponding to the failed joints are removed and where $w(J) = \sqrt{\det(JJ^T)}$ is the manipulability index of J [18]. This quantity is a local measure of the amount of dexterity that is retained when a manipulator suffers one or more locked-joint failures. The value of a relative manipulability index ranges from zero to one. A zero value would indicate a loss of full end-effector motion at that configuration after the failed joints are locked. In other words, a zero relative manipulability index means that the reduced manipulator Jacobian ${}^{i_1\cdots i_f}J$ does not have full rank. A relative manipulability index of one would indicate that no dexterity is lost at that configuration. In this case the joints in question do not contribute to end-effector motion at the operating configuration prior to their failure, i.e., those joints only produce self-motion [12], [19].

The inverse Jacobian equation for a Gough-Stewart platform (GSP) is given by

$$\dot{\mathbf{q}} = M\mathbf{v}, \tag{3}$$

which has a similar structure as (1). A GSP is a parallel mechanism consisting of a base, a moving platform, and struts. For a GSP, the inverse Jacobian M maps the generalized velocity of the payload to the corresponding joint velocities of the individual struts. The matrix M has the same form as the transpose of a manipulator Jacobian J. In other words, the first three components of each row forms a unit vector that is orthogonal to the vector given by the last three components of that row.

The types of failures for the GSP considered here are called hard failures and torque failures [10]. A *hard failure* is caused by mechanical fatigue or blown-off struts. When this occurs, the system acts as if the failed struts are totally lost. In this case, not only is actuation lost, but so are the mechanical constraints implemented by the strut. A *torque failure*, also known as a free swinging failure [3], refers to a hardware or software fault in a robotic manipulator that causes the loss of torque (or force) on a joint. Examples include a ruptured seal on a hydraulic actuator, the loss of electric power and brakes on an electric actuator, and a mechanical failure in a drive system. A joint with torque failure can move passively. Like a hard failure, a torque failure can be tolerated by designing the original system to be kinematically redundant. Similar to the case of a serial manipulator, the kinematic equations for a GSP following a hard or torque failure are obtained by removing the corresponding row of M. In [17], a class of GSPs was identified that possess optimal fault-tolerant manipulability for single joint failures based on maximizing the minimum relative manipulability index about an operating point.

Our analysis is applicable to serial and parallel mechanisms with the types of failures described above, so throughout this work we will use M and J^T interchangeably. Let J be a full rank $m \times n$ matrix with $m < n$ and let $r = n - m$. For a manipulator, m denotes the dimension of the workspace, n denotes the number of joints, and r denotes the degree of redundancy. We will call an $n \times r$ matrix N a *null space matrix* of J if the columns of N form an orthonormal basis for the null space of J. Although the null space matrix N is not unique for a given J, any two null space matrices N and N' of J are related by an orthogonal matrix Q in the following way: $N' = NQ$. We will see later that we can use Q to place N into a canonical form that can help us to properly view the null space and its relationship to fault tolerance.

In [12], it was shown that the relative manipulability index is related to the null space matrix by the relationship

$$\rho_{i_1,\cdots,i_f} = w(N_{i_1\cdots i_f}) = \sqrt{|N_{i_1\cdots i_f} N_{i_1\cdots i_f}^T|}, \tag{4}$$

where $N_{i_1\cdots i_f}$ is the $f \times r$ matrix consisting of rows $i_1, \ldots, i_f$ of the matrix N. We thus have the interesting observation that the relative manipulability indices are strictly a function of the null space of J. This important fact motivates the problem of designing operating configurations for robotic mechanisms based on choosing the manipulator Jacobian to have a prescribed null space. We will build on this result to address the issue of designing manipulators that are optimally fault-tolerant to one or more joint failures.

III. BOUNDS ON THE MINIMUM RELATIVE MANIPULABILITY INDEX

The relative manipulability index squared, $\rho^2_{i_1,\cdots,i_f} = |N_{i_1\cdots i_f} N^T_{i_1\cdots i_f}|$, is perhaps best viewed as a principal minor of the null space projection operator $P_N = I - J^+ J$, where J^+ denotes the pseudoinverse of J. The $n \times n$ matrix P_N represents the orthogonal projection of the joint space onto the null space of J. Unlike a null space matrix, P_N is unique for a given J; however, given a corresponding null space matrix N, we have $P_N = NN^T$. It then follows from (4) that the relative manipulability index squared is equal to the determinant of the submatrix consisting of the $i_1, \ldots, i_f$ rows and columns of P_N.

Recall that a $k \times k$ *minor* of an $n \times n$ matrix $A = [a_{ij}]$ with $k < n$ is a subdeterminant of the form

$$A\begin{pmatrix} i_1 & \cdots & i_k \\ j_1 & \cdots & j_k \end{pmatrix} \triangleq \begin{vmatrix} a_{i_1j_1} & a_{i_1j_2} & \cdots & a_{i_1j_k} \\ a_{i_2j_1} & a_{i_2j_2} & \cdots & a_{i_2j_k} \\ \vdots & \vdots & \ddots & \vdots \\ a_{i_kj_1} & a_{i_kj_2} & \cdots & a_{i_kj_k} \end{vmatrix}, \tag{5}$$

where $1 \le i_1 < \cdots < i_k \le n$ and $1 \le j_1 < \cdots < j_k \le n$. If $(j_1, \ldots, j_k) = (i_1, \ldots, i_k)$, then this quantity is called a *principal minor* of A. Hence, we have that $\rho^2_{i_1,\cdots,i_f}$ is the $(i_1, \ldots, i_f)$ principal minor of $P_N = NN^T$:

$$\rho^2_{i_1,\cdots,i_f} = P_N\begin{pmatrix} i_1 & \cdots & i_f \\ i_1 & \cdots & i_f \end{pmatrix}. \tag{6}$$

It is well known that the coefficients of the characteristic polynomial $p_A(\lambda) = |\lambda I - A|$ of A are given in terms of the sums of the principal minors of A. To be more specific, for $p_A(\lambda) = \lambda^n + a_{n-1}\lambda^{n-1} + \cdots + a_0$, we have that

$$a_{n-k} = (-1)^k \sum_{1\le i_1<\cdots<i_k\le n} A\begin{pmatrix} i_1\cdots i_k \\ i_1\cdots i_k \end{pmatrix}. \tag{7}$$

Since P_N is a projection, it is idempotent, i.e., $P_N^2 = P_N$, so its only possible distinct eigenvalues are 0 and 1. Furthermore, because $\mathrm{rank}(P_N) = r < n$ where $r = n - m$, it follows that the characteristic polynomial of P_N is

$$p(\lambda) = \lambda^m(\lambda - 1)^r = \sum_{k=0}^{r} \binom{r}{k} (-1)^k \lambda^{n-k}. \tag{8}$$

Equations (6), (7), and (8) then imply that

$$\sum_{1\le i_1<\cdots<i_f\le n} \rho^2_{i_1,\cdots,i_f} = \binom{r}{f}. \tag{9}$$

This result, written as a slightly different but equivalent expression, was also proved in [17]; however, the proof provided there was based on repeated application of the Binet-Cauchy theorem and was less direct than applying principal minors. It is important to note, however, that the approach just given is not merely a different proof of the result in [17]. More importantly, it provides us with an approach that will be used in Section IV to address multiple joint failures.

As noted in [17], equation (9) can be used to obtain an upper bound for the worst case relative manipulability index by noting that the minimum value of any set of numbers must be less than or equal to the average so that

$$\min_{1\leq i_1<\cdots<i_f\leq n} \rho_{i_1,\cdots,i_f} \leq \sqrt{\frac{\binom{r}{f}}{\binom{n}{f}}}. \tag{10}$$

IV. DESIGNING OPTIMALLY FAULT-TOLERANT MANIPULATOR JACOBIANS

Some parallel mechanism configurations such as the one shown in Fig. 1 are naturally fault-tolerant due to symmetry. In this particular example, it is clear that each strut has the same overall influence on the motion of the device. Consequently, this manipulator is equally fault-tolerant to any single joint failure. This can be verified analytically by examining the corresponding null space matrix for the 8-DOF manipulator shown in Fig. 1:

$$N = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & -1 \\ 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & -1 \end{bmatrix}. \tag{11}$$

It can be observed that each row of N has the same length, and that the relative manipulability index ρ_i for each joint is equal to 0.5. In light of the bound given in (10), this configuration is optimally fault-tolerant in terms of maximizing the minimum relative manipulability index ρ_i. However, it is equally easy to see that this design can be fault intolerant to two joint failures. From (11), it is clear that locking the joints corresponding to two even numbered struts or to two odd numbered struts results in a reduced Jacobian that is singular, e.g., $\rho_{13} = 0$. Thus, additional care must be given if one wants to address fault tolerance to multiple failures.

Equation (10) served as a motivation in [17] for defining a manipulator operating about a single point in the workspace to be optimally fault-tolerant to $f \leq r$ failures if all of its relative manipulability indices $\rho_{i_1,\cdots,i_f}$ are equal, i.e.,

$$\rho_{i_1,\cdots,i_f} = \sqrt{\frac{\binom{r}{f}}{\binom{n}{f}}} \tag{12}$$

for $1 \leq i_1 < \cdots < i_f \leq n$. In this article, we will prefer to say that a manipulator is *equally fault-tolerant to $f \leq r$ failures* at an operating configuration if (12) holds for $1 \leq i_1 < \cdots < i_f \leq n$ at that configuration. Note that equal fault tolerance is a local property since it would apply to specific configurations and would be most applicable for manipulators operating in a small workspace. If a manipulator is equally fault-tolerant to $f \leq r$ failures, then by (10) it is optimally fault-tolerant in a worst case relative manipulability index sense to $f \leq r$ failures. However, while it is clear that an optimal value exists, it is possible that a manipulator may not have a configuration that is equally fault-tolerant to f failures. In this case, the optimal value is smaller than the bound given in (10). It is the goal of this section to show that this is typically the case.

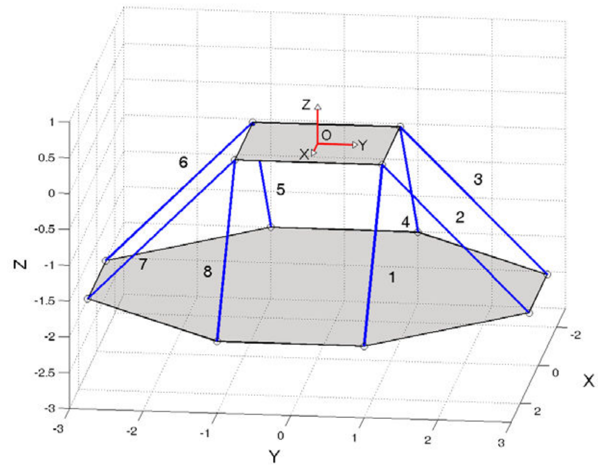

Fig. 1. A symmetric 8-DOF GSP. The geometric symmetry ensures enough algebraic symmetry in the manipulator Jacobian and the null space matrix that optimal fault tolerance is guaranteed for single joint failures. However, in spite of having two degrees of redundancy, the manipulator is fault intolerant to certain combinations of failures in two joints.

In order to study equally fault-tolerant configurations, we use the following result, which was proved in [20]:

Theorem 1: If a manipulator is equally fault-tolerant to f failures where $1 < f \leq r$, then it is also equally fault-tolerant to $f-1$ failures. Furthermore, the manipulator is equally fault-tolerant to k failures for $k = 1, 2, \ldots, f$.

Theorem 1 has some important implications. This follows from the fact that it forces P_N to have a particularly simple structure when the manipulator is equally fault-tolerant to more than one failure. To see this, first note that if J is equally fault-tolerant to a single failure, then the diagonal elements of P_N are all equal to r/n. If J is equally fault-tolerant to $f \geq 2$, then by Theorem 1 it is equally fault-tolerant to single failures and to two failures. Hence, the (i,j) principal minor of the symmetric matrix P_N is

$$\begin{vmatrix} r/n & p_{ij} \\ p_{ji} & r/n \end{vmatrix} = \frac{r^2}{n^2} - p_{ij}^2 = \frac{r(r-1)}{n(n-1)}, \tag{13}$$

where we have used the fact that $p_{ji} = p_{ij}$, and where the last equality follows from the assumption of equal fault tolerance to two failures. Solving for p_{ij} gives $p_{ij} = \frac{\pm 1}{n}\sqrt{\frac{r(n-r)}{n-1}}$ for all $1 \leq i < j \leq n$. Hence, when J is equally fault-tolerant to $f \geq 2$ failures, the diagonal elements of P_N are all equal and the off-diagonal elements of P_N all have the same magnitude. To simplify matters further, note that multiplying any of the columns of J by -1 does not affect the fault tolerance properties of J. In doing so, the corresponding rows and columns of P_N are also multiplied by -1 so that we can assume without loss of generality that P_N has the form

$$P_N = \begin{bmatrix} a & b & b & \cdots & b \\ b & a & \pm b & \cdots & \pm b \\ b & \pm b & a & \cdots & \pm b \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ b & \pm b & \pm b & \cdots & a \end{bmatrix} \tag{14}$$

where $a = \frac{r}{n}$ and $b = \frac{-1}{n}\sqrt{\frac{r(n-r)}{n-1}}$. We use the property that P_N is a projection to determine restrictions on the number of degrees of redundancy that a fully spatial manipulator can have for the equal fault tolerance property to hold. As a projection, $P_N^2 = P_N$ so that for $j > 1$,

$$b = p_{1j} = (P_N)_{1j} = (P_N^2)_{1j} = 2ab + qb^2, \tag{15}$$

where q is an integer given by $n_1 - n_2 - 1$, where n_1 denotes the number of elements in the j-th column of P_N that are equal to b and n_2 denotes the number of elements equal to $-b$. Clearly $n_1 + n_2 = n - 1$ as $(P_N)_{jj} = a$ and $(P_N)_{ij} = \pm b$ for $i \neq j$. Since $b \neq 0$, (15) yields

$$q = \frac{1 - 2a}{b}. \tag{16}$$

For a fully spatial redundant manipulator, $m = 6$ and $n = r + 6$. Substituting the expressions for a and b into (16) gives

$$q = \frac{1 - \frac{2r}{n}}{\frac{-1}{n}\sqrt{\frac{r(n-r)}{n-1}}} = (r - m)\sqrt{\frac{r + m - 1}{mr}}. \tag{17}$$

The requirement that q is an integer is a necessary condition for the existence of a manipulator having $r > 1$ degrees of redundancy with the property that it is equally fault-tolerant to two failures.

Unfortunately, the requirement that q is an integer automatically eliminates most spatial manipulator designs since only specific values of r are feasible. Indeed, it was shown in [20] that regardless of a manipulator's geometry or the amount of kinematic redundancy present in a manipulator, no fully spatial manipulator Jacobian can be equally fault-tolerant to two joint failures. In this article, we will examine the cases when the workspace has dimension $m = 2$ and 3.

We begin by identifying those positive integers r such that the resulting q is an integer. We will do this by first solving a simpler problem. If q is an integer, then so is

$$mq^2 = r^2 - (m+1)r - m(m-2) + \frac{m^2(m-1)}{r}. \tag{18}$$

Since the first three terms in the expansion of mq^2 are integers, so is the last term, $m^2(m-1)/r$, i.e., r divides $m^2(m-1)$.

In the case of a fully spatial manipulator, $m = 6$. There are exactly 18 positive integers that divide $m^2(m-1) = 180 = 2^2 \cdot 3^2 \cdot 5$, each having the form $r = 2^i 3^j 5^k$ with $i, j = 0, 1, 2$ and $k = 0, 1$. Those positive integers r for which q is an integer are among these 18 candidates. Testing all 18, we find that q is an integer only for $r = 1$, 3, 6, and 10. We are not interested in the case $r = 1$ since the manipulator could only be fault-tolerant to single joint failures (in this case the manipulator is equally fault-tolerant to two failures in the undesirable sense that the relative manipulability index is zero for any combination of two distinct failures). The remaining candidate values of r can be eliminated based on the requirements of the row structure of N. Details can be found in [20].

For $m = 2$, we have that r divides 4 so that $r = 2$ or 4. These two possibilities correspond to $q = 0$ and $\sqrt{5/2}$, respectively. As q is an integer, we are left with $r = 2$ as the only possible candidate for $m = 2$. Unfortunately, a careful examination of the structure of the corresponding null space matrix excludes this candidate as well. Thus we have the result that no full rank $2 \times n$ Jacobian with $n \geq 4$ can be equally fault-tolerant to two failures. In particular, no planar nR manipulator configuration can be equally fault-tolerant to two or more locked-joint failures.

Lastly, we consider the case $m = 3$. In this case, $m^2(m-1) = 18$ so that the candidate values for r are 2, 3, 6, 9, and 18. The requirement that q is an integer further reduces the candidate list to $r = 3$ and 6. Furthermore, the required structure of the null space matrix eliminates the case $r = 6$. It turns out that there are suitable null space matrices for $r = 3$. A particular example is

$$N = \begin{bmatrix} 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & \sqrt{\frac{2}{5}} & \frac{1}{\sqrt{10}} \\ -\sqrt{\frac{5+\sqrt{5}}{20}} & \frac{\sqrt{5}-1}{2\sqrt{10}} & \frac{1}{\sqrt{10}} \\ -\sqrt{\frac{5-\sqrt{5}}{20}} & \frac{-\sqrt{5}-1}{2\sqrt{10}} & \frac{1}{\sqrt{10}} \\ \sqrt{\frac{5-\sqrt{5}}{20}} & \frac{-\sqrt{5}-1}{2\sqrt{10}} & \frac{1}{\sqrt{10}} \\ \sqrt{\frac{5+\sqrt{5}}{20}} & \frac{\sqrt{5}-1}{2\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix} \tag{19}$$

In this case, there is a nice geometric interpretation for the rows of N, which correspond to points on a sphere of radius $1/\sqrt{2}$ with the first row of N corresponding to the North Pole and the remaining five rows corresponding to the vertices of a pentagon located at a latitude of $90° - \arccos(1/\sqrt{5})$. The null space matrix (19) determines a family of equally fault-tolerant 3×6 positional Jacobians. A particular example is

$$J = \begin{bmatrix} 0.7071 & 0.0000 & 0.0000 \\ -0.3162 & 0.0557 & -0.6300 \\ -0.3162 & 0.3253 & 0.5424 \\ -0.3162 & -0.5820 & -0.2476 \\ -0.3162 & 0.6164 & -0.1418 \\ -0.3162 & -0.4153 & 0.4770 \end{bmatrix}^T. \tag{20}$$

Other Jacobians can be generated from (19). For example, one can permute the rows of N or multiply one or more of the rows by -1. This will not affect fault tolerance, but it will generally result in a different Jacobian.

V. CONCLUSIONS

Designing kinematically redundant manipulators that are optimally fault-tolerant to multiple joint failures is an important problem for tasks requiring robots to operate in remote and hazardous environments. In previous work, it was shown that no manipulator configuration can be equally fault-tolerant to three or more failures and that no fully spatial manipulator can be equally fault-tolerant to two locked-joint failures. However, the case of a two or three dimensional workspace was not addressed. In this work, it was shown that no manipulator working in a two-dimensional workspace can be equally fault-tolerant to two or more failures while one can design a family of Jacobians for a six degree-of-freedom manipulator operating in a three-dimensional workspace that are equally fault-tolerant to two simultaneous locked-joint failures.

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