

# **Prime Prime Bar:**

*An Introduction to the  
Planetary Boundary Layer and Cumulus Convection*

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