Isentropic slopes, down-gradient eddy fluxes, and the extratropical atmospheric circulation response to tropical tropospheric heating

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Abstract

Climate change experiments run on IPCC-class numerical models consistently suggest that increasing concentrations of greenhouse gases will lead to a poleward shift of the mid-latitude jets and their associated eddy fluxes of heat and potential vorticity (PV). Experiments run on idealized models suggest that the poleward contraction of the jets can be traced to the effects of increased latent heating and thus locally enhanced warming in the tropical troposphere. Here we provide new insights into the dynamics of the circulation response to tropical tropospheric heating using transient experiments in an idealized general circulation model.

It is argued that the response of the mid-latitude jets to tropical heating is driven fundamentally by: 1) the projection of the heating onto the meridional slope of the lower tropospheric isentropic surfaces; and 2) a diffusive model of the eddy fluxes of heat and PV. In the lower and middle troposphere, regions where the meridional slope of the isentropes (i.e., the baroclinicity) is increased are marked by anomalously poleward eddy fluxes of heat, and vice versa. Near the tropopause, regions where the meridional gradients in PV are increased are characterized by anomalously equatorward eddy fluxes of PV, and vice versa. The barotropic component of the response is shown to be closely approximated by the changes in the lower level heat fluxes. As such, the changes in the eddy fluxes of momentum near the tropopause appear to be driven primarily by the changes in wave generation in the lower troposphere.
1. Introduction

Anthropogenic emissions of greenhouse gases are linked in a range of numerical experiments to a robust poleward contraction of the mid-latitude jets and their associated eddy fluxes of heat and momentum (Fyfe et al. 1999; Kushner et al. 2001; Yin 2005; Miller et al. 2006; Arblaster and Meehl 2006; Hegerl et al. 2007; Lu et al. 2008; Schneider et al. 2010). The poleward shift of the jets projects onto annular modes of climate variability (e.g., Kushner et al. 2001; Miller et al. 2006) and thus has pronounced implications for climate change throughout the extratropics. The simulated response of the mid-latitude jets to increasing carbon dioxide is among the most robust circulation responses found in climate change experiments run on fully coupled, IPCC-class models (Hegerl et al. 2007).

The amplification of tropospheric warming by increased latent heating in the tropics has been hypothesized to be a key factor in driving the simulated trends in the mid-latitude jets. For example: Lim and Simmonds (2008) demonstrate that the latitude of the Southern Hemisphere mid-latitude jet is sensitive to tropical heating in their full-physics atmospheric general circulation model, and Butler et al. (2010) demonstrate that the poleward contraction of the mid-latitude jets is a robust response to climate change-like heating in the tropical troposphere in an idealized atmospheric general circulation model. Chen and Held (2007) and Lu et al. (2008) point to the importance of changes in the meridional temperature gradient near the tropopause level in driving the shift of the jet in climate change simulations, albeit they do not explicitly link the shift in the jet to tropical heating.
The mechanisms whereby tropical heating drives a poleward shift in the mid-latitude jets remain unclear. Tropical heating is expected to drive an eastward acceleration of the flow in the mid-latitude upper troposphere/lower stratosphere via thermal wind balance. But the response of the thermal wind does not explain the simulated changes in the eddy fluxes of heat and momentum that accompany the poleward shift in the jet. The eddy response has been interpreted in the context of changes in subtropical static stability (Frierson 2008; Lu et al. 2008), changes in the height of the tropopause (Williams 2006; Lorenz and DeWeaver 2007), changes in the surface temperature gradient (Brayshaw et al. 2008; Lu et al. 2010; Chen et al. 2010), changes in the eddy length scale (Kidston et al. 2011), and the influence of the thermally driven eastward wind anomalies on the Rossby-wave phase speed (Chen and Held 2007; Chen et al. 2008). In the latter case, the increase in the Rossby-wave phase speed leads to a poleward shift in the region of maximum wave breaking in the subtropics, and through the adjustment of the meridional circulation, a poleward shift in the low-level baroclinic zone (e.g., Robinson 2000).

The purpose of this study is to provide an alternative perspective on the response of the mid-latitude tropospheric circulation to zonal-mean tropical heating. The response is interpreted in the context of a) the projection of the heating onto the climatological-mean isentropic surfaces and thus the influence of the heating on the meridional slope of the lower tropospheric isentropic surfaces; and b) a diffusive model of the eddy fluxes of heat near the surface and potential vorticity (PV) near the tropopause. In the following section we outline the premise for our guiding hypothesis. Section 3 reviews the
experiment design and analysis details. Results are given in Section 4 and are discussed in Section 5.

2. Guiding hypothesis

The role of the eddy fluxes in the atmospheric general circulation can be approached in two ways:

1) Eddies are treated as (Rossby) waves. They are generated near the surface in regions of large baroclinicity, their propagation is determined by the index of refraction, and they dissipate in regions determined by wave properties such as wavenumber and phase speed (e.g., Matsuno 1970; Chen and Robinson 1992).

2) Eddies are treated as (macro)turbulence. They act fundamentally to diffuse heat near the surface in regions of large baroclinicity (e.g., Kushner and Held 1998; Held 1999; Held and Schneider 1999; Schneider 2004) and PV in the free atmosphere in regions of large PV gradients (e.g., Green 1970; Held 1999).

Potential vorticity is intimately tied to both perspectives. Rossby waves owe their existence to basic state PV gradients and the refractive index is a function of the PV gradient. Eddy turbulence is described by diffusive eddy fluxes of PV and these are often parameterized to be proportional to the basic state PV gradient. But in the wave perspective, the eddy fluxes of heat and PV are determined by waveguides, phase speeds and other wave properties. In the (macro)turbulence perspective, the eddy fluxes are determined primarily by the eddy diffusivities. Nevertheless, both wave and turbulence perspectives are closely related: breaking Rossby waves lead to turbulent eddy fluxes and
these in turn form and maintain waveguides through self-sharpening of basic state PV gradients (e.g., McIntyre 2008).

The diffusive model provides an excellent approximation of the eddy fluxes of heat near the surface where radiation plays a dominant role in setting the mean temperature gradients (e.g., Kushner and Held 1998; Held 1999). The interpretation of the diffusive model is more nuanced in the free atmosphere where the eddies play a more prominent role in determining the mean gradients in PV (Held 1999; see also the Discussion section in this paper). Nevertheless, as demonstrated here, the diffusive model of the PV fluxes provides a seemingly robust qualitative estimate for the changes in the eddy fluxes of PV driven by diabatic heating.

The guiding hypothesis of our analysis is the following:

a) Diabatic heating in the tropical troposphere has two direct effects on the extratropical circulation: 1) the heating adjusts the meridional slope of the isentropes in the free troposphere; and 2) the heating adjusts the distribution of PV at the tropopause via its influence on atmospheric static stability (i.e., the thickness of the isentropic layers).

b) The eddy fluxes of heat and PV adjust to the direct effects of the heating in a manner consistent with the flux-gradient relationship. At lower levels, the poleward eddy fluxes of heat are increased in regions where the isentropic slopes are anomalously steep (i.e., the baroclinicity is increased), and decreased where the isentropic slopes are flattened. At upper levels, the equatorward eddy fluxes of PV are increased where the pole-equator gradients in PV are enhanced, and decreased where the gradients are diminished. Note that in the diffusive model, the eddy fluxes are a function of not only
the mean gradients but also the eddy diffusivities. As discussed in Section 5, our
approach implicitly assumes that the diffusivities remain largely unchanged in response
to the heating.

In the following section we outline the experiment design and provide details of
the analyses. Results are presented in Section 4.

3. Experiment design and analysis details

a) Experiment design

We test the above hypothesis by examining the transient response to tropical
heating in the dry dynamical core of the Colorado State University general circulation
model (Ringler et al. 2000). The model is identical to that used in Butler et al. (2010).
The vertical coordinate system is hybrid sigma-isentropic (Konor and Arakawa 1997) and
transitions from sigma near the surface to isentropic in the free troposphere and
stratosphere. There is no topography, no ocean, and there are 25 vertical layers with a
model top at 1 hPa. The model is discretized on a geodesic grid (Heikes and Randall
1995) with 10,242 grid cells (a horizontal resolution of ~250 kilometers).

All experiments are run with Held-Suarez (1994) parameterizations, and the
circulation is driven by Newtonian relaxation towards an equinoctial radiative
equilibrium temperature profile. The impact of using a wintertime radiative equilibrium
profile was tested in Butler et al. (2010) and found to have only a small qualitative effect
on the atmospheric response to tropical tropospheric heating.

The heating is given in Butler et al. (2010; c.f. Figure 2a and Table 1) and is
reproduced here in Figure 1. The forcing is zonally-symmetric, has peak amplitude of 0.5
K/day and is centered at ~300 hPa and the equator. Changes in the depth, altitude, and shape of the tropical thermal forcing affect the amplitude of the extratropical response (Butler et al. 2010; c.f., Figure 2). But as discussed further in Section 5 (and demonstrated in Butler et al. 2010), the poleward shift of the middle latitude jet is reproducible when the heating is contracted in the vertical and meridional directions. The weak projection of the heating in the lower tropical stratosphere does not affect the results in a notable manner (Butler et al. 2010; c.f., Figure 2).

Since we will be examining the response in terms of the isentropic slope and the eddy flux of PV on isentropic surfaces, the heating is superposed on the model isentropes in Figure 1a, and is transposed into an isentropic coordinate system in Figure 1b. Note that the isentropic surfaces in the lower-middle troposphere tilt upwards with latitude (Figure 1a). Thus for the thermal forcing considered here, the isentropes below ~320 K experience peak heating not at the Equator but at subtropical latitudes (Figure 1b). The solid black lines in Figures 1a and 1b denote the dynamical tropopause, defined here as the level at which PV is equal to two PV units (one PVU=10^{-6} \text{m}^2 \text{s}^{-1} \text{K} \text{kg}^{-1}).

The steady-state response to the heating was explored in Butler et al. (2010). Here we consider the transient response. The transient simulations are started from 12 different initial conditions generated from a control run that has been spun-up for 360 days. The initial conditions are separated by 50 days to ensure independence. In the forced experiments, the thermal forcing is turned on after 10 days, and each member run lasts 150 days. In the unforced experiments, the thermal forcing remains off at all time steps. The use of an equinoctial equilibrium temperature profile provides twice the sample size
assuming the responses in the two hemispheres are independent of each other. Thus there are 24 ensemble members in the forced and unforced experiments.

In all figures, the "response" is defined as the difference between the ensemble-mean of the 24 forced runs and the ensemble-mean of the 24 unforced runs. The statistical significance of key aspects of the steady-state response is established in Butler et al. 2010 (Tables 2 and 4). All key results highlighted here (i.e., changes in the zonal wind, PV, isentropic slope, and the eddy fluxes of heat and PV) are qualitatively reproducible in all 24 ensemble members.

\textit{b) Analysis details}

The response to the thermal forcing is examined in the context of the isentropic slope and the eddy fluxes of heat and PV. The eddy fluxes of heat are examined on pressure levels in the lower troposphere since isentropic surfaces intersect the surface. Potential vorticity and the eddy fluxes of PV are examined on isentropic surfaces at and above 300 K. Potential vorticity is defined on isentropic surfaces as:

\begin{equation}
\begin{aligned}
P &= \frac{(f + \zeta)}{\sigma} \\
\end{aligned}
\end{equation}

where the isentropic density (the inverse of the “thickness” of the isentropic layer) is:

\begin{equation}
\begin{aligned}
\sigma &= -\frac{1}{g} \frac{\partial p}{\partial \theta}. \\
\end{aligned}
\end{equation}
In (1) and (2), (capital) $P$ is the potential vorticity, $f$ is the planetary vorticity, $\zeta$ is the relative vorticity calculated on isentropic surfaces, $g$ is the gravitational constant, $\theta$ is potential temperature, and (lower case) $p$ is pressure.

The eddy fluxes of PV in isentropic coordinates are mass-weighted, and are calculated in the following manner: First, the mass-weighted zonal-mean meridional wind ($v$) and potential vorticity ($P$) are found as:

$$
\bar{v}^* = \frac{\sigma v}{\sigma} \quad \text{and} \quad \bar{P}^* = \frac{\sigma P}{\sigma}
$$

where the overbars denote the zonal mean and the stars denote mass-weighted zonal-mean quantities. The eddy components of the mass-weighted meridional wind and potential vorticity fields are subsequently found as:

$$
\hat{v} = v - \bar{v}^* \quad \text{and} \quad \hat{P} = P - \bar{P}^*
$$

Where hats denote deviations from the mass-weighted zonal-mean. Finally, the mass-weighted zonal-mean eddy fluxes of PV (in units of $10^{-6} \text{ m}^3 \text{ s}^{-2} \text{ K kg}^{-1}$ or - equivalently - PVU m s$^{-1}$) are calculated as:

$$
\bar{\nu}^* \bar{P} = \frac{\sigma \hat{v} \hat{P}}{\sigma}
$$
In isentropic coordinates, the eddy PV flux is directly related to the convergence of the Eliassen-Palm (EP) flux - and thus to the eddy fluxes of thickness (i.e., heat) and momentum - as:

\[ \frac{\sigma \hat{v} \hat{P}}{\sigma} \equiv \frac{1}{\sigma} \nabla \cdot F \equiv \frac{1}{\sigma} \left\{ -\nabla' \frac{\partial}{\partial \varphi'} u' - \frac{1}{\sigma} (f - \frac{\partial}{\partial \varphi'} u) \sigma' \right\} \]

where \( \nabla \cdot F \) is the divergence of the EP flux vector; \( u \) is the zonal wind; \( a \) is the radius of the Earth; and \( \varphi \) is latitude (e.g., Tung 1986, eqn. 4.3). The first term on the RHS corresponds to the meridional divergence of the eddy flux of zonal momentum. The second term on the RHS corresponds to the vertical divergence of the eddy flux of heat. Recall that in isentropic coordinates, the eddy fluxes of heat correspond to fluxes of thickness, and are thus analogous to form drag.

4. Results

Figure 2 shows the response of zonal-mean temperature (upper panels) and zonal wind (lower panels) to the heating averaged over days 100-150 of the integration. The results are shown in pressure (left) and isentropic coordinates (right). Note that the temperature response is converted to pressure when shown in isentropic coordinates (i.e., warming corresponds to the movement of isentropic surfaces to higher pressures). The temperature response (top) includes warming that spans the entire tropics and cooling in the lower polar stratosphere. The polar stratospheric cooling results from dynamical changes in the stratospheric residual circulation (Butler et al. 2010) and is not examined.
here. Note that due to the meridional slope of the mean isentropic surfaces, the pressure response below ~320 K is largest not at the Equator but at subtropical latitudes (Figure 2b).

Figure 3 highlights the time varying response to the heating in the zonal wind at 350 K (top) and 700 hPa (bottom). The response in the upper troposphere/lower stratosphere (e.g., ~350 K) is eastward throughout the middle latitudes (Figures 2d, 3a). In contrast, the response in the lower troposphere (e.g., ~700 hPa) is characterized by a dipole in the zonal wind field, with eastward anomalies located between ~50-70 degrees and westward anomalies between ~30-40 degrees (Figure 2c, 3b). Interestingly, the changes in the near-surface wind field propagate poleward by ~5 degrees throughout the 150 days of the transient integration (Figure 3b). The response is near equilibrium by day ~100 of the integration (Figure 3) and thus the response averaged over days 100-150 bears strong resemblance to the steady-state response to the same heating (i.e., compare Figure 2a and 2c from this study with Figure 2a from Butler et al. 2010).

As demonstrated in Butler et al. (2010), the response to the heating is consistent with a poleward shift in the model mid-latitude tropospheric jet. The dipole in the tropospheric wind field reflects a poleward shift in the latitude of maximum surface westerlies, and is accompanied by three statistically significant changes in the eddy fluxes (Butler et al. 2010; c.f., Figure 2a): 1) anomalously poleward eddy fluxes of zonal momentum in the upper troposphere across ~50 degrees; 2) anomalously poleward eddy heat fluxes in the lower troposphere poleward of ~40 degrees; and 3) anomalously equatorward eddy heat fluxes in the lower troposphere equatorward of ~40 degrees. Below we first consider the response in terms of the eddy fluxes of heat in the free
troposphere. We then consider the response of the eddy fluxes of PV at the tropopause. The barotropic component of the response (i.e., the component driven by the eddy fluxes of momentum) is considered in the Discussion section.

Eddy fluxes of heat in the troposphere

In a baroclinic atmosphere, the climatological-mean pressure and isentropic surfaces are not parallel. Thus a heating profile that varies monotonically along pressure surfaces will, in general, not vary monotonically along isentropic surfaces. The projection of the heating onto the climatological-mean isentropic surfaces is important, as the projection determines the effect of the heating on the meridional slope of the isentropic surfaces. The meridional slope of the isentropic surfaces - in turn - plays a key role in the development of baroclinic waves: it is equivalent to the baroclinicity of the flow and is implicitly included in metrics of instability such as Eady growth rate (i.e., the Eady growth rate is proportional to the ratio of the meridional gradient and square root of the vertical gradient in temperature; Lindzen and Farrell 1980).

The initial response of the meridional slope of the isentropes when the tropical heating (in terms of potential temperature tendency, $\dot{\theta}$) is turned on can be expressed analytically as follows. First, define the meridional slopes of the isentropic surfaces as:

$$ s_\theta = -\frac{\partial \theta / \partial y}{\partial \theta / \partial z} $$

Where $\theta$ denotes the potential temperature. Similarly, define the meridional slope of the tropical heating as:
\[
(8) \quad s_\tilde{\theta} = -\frac{\partial \tilde{Q}}{\partial \tilde{y}} \frac{1}{\partial Q / \partial \tilde{z}}
\]

The time rate of change of the meridional slope of the isentropic surfaces when the heating is turned on is given by the derivative of (7) with respect to time:

\[
(9) \quad \frac{\partial s_\theta}{\partial t} = -\frac{\partial \tilde{Q}}{\partial \theta} \frac{1}{\partial \theta / \partial \tilde{z}} - s_\theta \frac{\partial \tilde{Q}}{\partial \theta / \partial \tilde{z}} = (s_\tilde{\theta} - s_\theta) \frac{\partial \tilde{Q}}{\partial \theta / \partial \tilde{z}}
\]

Where we have assumed that when the heating is initially turned on, \( \partial \theta / \partial t = \tilde{Q} \).

Equation (9) makes clear that the initial rate of change of the isentropic slopes is a function of the meridional and vertical structure of both the heating (i.e., the thermal forcing) and the isentropic surfaces. If the meridional slope of the heating \( s_\tilde{\theta} \) is less than that of the isentropic surfaces \( s_\theta \), then \( s_\theta \) will decrease where the heating grows with height \( (\partial \tilde{Q}/\partial z > 0) \) and increase where the heating damps with height \( (\partial \tilde{Q}/\partial z < 0) \).

Equation (9) also makes clear that the heating will always change the isentropic slope except where (i) the slopes of the heating and the isentropes are identical (i.e., if \( s_\tilde{\theta} - s_\theta = 0 \)) or (ii) the vertical gradient of the heating is zero. These conditions are possible locally but not globally unless (i) the heating extends from pole-to-pole or (ii) the heating is uniform with height.

In the case of the tropical heating considered here, \( \tilde{Q} \) decreases uniformly with latitude on pressure surfaces (Figure 1a). Since the climatological-mean isentropic
surfaces below ~320 K slope upwards with latitude (Figure 1a, solid contours), it follows that such isentropes will experience largest heating (and hence largest pressure rises) not at the Equator, but in the subtropics (Figures 1b and 2b). Thus the isentropic slope will be reduced in the subtropics and increased at middle latitudes. In the context of Equation (9), \( \partial \tilde{Q}/\partial z > 0 \) and \( (s_q - s_o) < 0 \) below the level of maximum heating in the subtropics whereas \( \partial \tilde{Q}/\partial z > 0 \) and \( (s_q - s_o) > 0 \) below the level of maximum heating in middle latitudes.

For example, consider the response of the isentropic slope at 310 K. The top panel in Figure 4 shows the projection of the thermal forcing on the 310 K surface (contours) and the response in pressure on the 310 K surface (shading) as a function of time and latitude. The thermal forcing projects most strongly onto the 310 K isentropic surface not at the Equator but ~40-50 degrees (Figure 4a contours; see also Figure 2b). As the 310 K surface is heated, it moves to higher pressure in approximately the same latitude band spanned by the heating. The projection of the heating onto the 310 K surface changes as the isentropic surface moves with time, and is most clearly aligned with the changes in pressure after day ~75.

The contours in the bottom panel of Figure 4 show the changes in the meridional slope of the 310 K surface (i.e., \( s_o \) at 310 K) in m/km; the shading shows the response in the eddy heat fluxes at 700 hPa. As the 310 K isentrope moves to higher pressure ~40-50 degrees latitude (Figure 4 top), the meridional slope of the 310 K surface is reduced in the subtropics and increased in the extratropics (Figure 4 bottom). Thus, baroclinicity is decreased equatorward of ~40 degrees but increased poleward of ~40 degrees. The eddy fluxes of heat clearly respond in a manner consistent with forcing by the changes in the
isentropic slope (Figure 4 bottom): regions where the meridional slope is increased are marked by anomalous poleward eddy fluxes of heat; regions where the meridional slope is decreased are marked by anomalous equatorward eddy fluxes of heat. As the integration proceeds, the eddy fluxes of heat will also adjust the slope of the isentropic surfaces. But the results in Figure 4 (bottom) strongly suggest that the eddy fluxes are to first order responding to the changes in baroclinicity driven by the tropical heating.

The results in Figure 4 thus reveal two key aspects of the dynamical response to tropical heating: 1) The projection of the heating onto the initial isentropic surfaces determines the initial changes in the meridional slopes of the isentropic surfaces; and 2) the changes in the meridional slopes of the isentropes, in turn, lead to changes in the eddy fluxes of heat: regions where the isentropic slopes are flattened (where the flow is stabilized) are marked by anomalously equatorward eddy fluxes of heat; regions where the isentropic slopes are increased (where the flow is destabilized) are marked by anomalously poleward eddy fluxes of heat. The anomalous eddy fluxes of heat are thus down-gradient in the sense that they are oriented away from the latitude of maximum heating. Similar results are found for isentropic surfaces between 300-320 K and for the eddy heat fluxes at pressure levels lower than 700 hPa (not shown). Surfaces below 300 K intersect the ground in the subtropics, making the results there more difficult to interpret.

*Eddy fluxes of PV near the tropopause*

The results in Figure 4 suggest that two physical factors contribute to the lower tropospheric response to tropical heating: the effect of the tropical heating on the
meridional slope of lower tropospheric isentropic surfaces and the diffusive nature of the eddy heat fluxes. As shown below, a diffusive model of the eddy fluxes also provides a close approximation for the changes in the eddy fluxes of PV near the tropopause.

The top two panels in Figure 5 show the distribution of PV (contours) and the eddy fluxes of PV (shading) averaged over days 100-150 for the unforced (top) and forced (middle) experiments. In general, the eddy fluxes of PV are oriented along the axis of the dynamical tropopause (i.e., where PV~2 PVU) and peak ~60 degrees. There is a secondary maximum in the eddy fluxes of PV in the mid-latitude stratosphere, but the stratospheric response is not examined in this study. The primary differences between the top and middle panels lie near the subtropical tropopause, where the heating acts to noticeably decrease PV and lift both the dynamical tropopause and the region of largest eddy PV fluxes.

The differences between the upper panels are quantified in Figure 5c. The contours show the differences in PV; the shading shows the changes in the eddy fluxes of PV. The bold solid lines show the 2 PVU contour for the unforced (lower line) and forced (upper line) experiments. As noted above, the heating leads to widespread decreases in PV near the subtropical tropopause centered ~40-50 degrees. The decreases in PV are driven primarily by changes in static stability (not shown) and correspond to a lifting of the dynamical tropopause. The anomalous eddy fluxes of PV are down the anomalous gradients in the PV field: i.e., the fluxes are anomalously poleward in the vicinity of the tropopause between ~20-40 degrees and anomalously equatorward between ~50-60 degrees.

The pattern of anomalous eddy PV fluxes in Figure 5c corresponds to anomalous
divergence of the EP flux in the subtropics and convergence of the EP flux in mid-latitudes between ~320 K and 340 K (Eq. 6). Thus, the results indicate a poleward shift in the latitude of largest wave breaking at those levels. Since the anomalous eddy fluxes of PV are down the anomalous gradients in PV, it follows that they do not drive the anomalous potential vorticity anomalies. Rather, the anomalous eddy PV fluxes can be interpreted as responding to the changes in the mean distribution of PV.

At first glance, the wave forcing of the tropopause-level zonal-wind anomalies may appear counterintuitive: i.e., the anomalous zonal flow is eastward in regions of anomalous equatorward PV fluxes (anomalous EP flux convergence), and vice versa. However, the anomalous upper level PV fluxes are dominated not by the meridional convergence of the momentum flux (which determines the barotropic component of the response), but by the vertical derivative of the heat flux (second term on the RHS of Eq. 6; result not shown). The anomalous vertical derivative of the heat flux gives rise to anomalous PV fluxes that act to damp the upper-level zonal wind anomalies, much as the climatological-mean equatorward PV fluxes act to damp the climatological-mean eastward flow near the extratropical tropopause (as is also true in the observations; e.g., Green 1970). The response of the upper tropospheric zonal flow should be viewed not in terms of the in-situ wave forcing, but rather as the sum of the barotropic component of the response (which is determined by the meridional convergence of the momentum flux) and the thermal wind. The barotropic component of the response is discussed further in Section 5.

Figure 6 explores the evolution of the upper-level responses in PV (contours) and the eddy fluxes of PV (shading) as a function of latitude and time on two isentropic
surfaces: 326 K and 335 K. The surfaces are chosen because 1) they lie within the region of largest anomalous PV fluxes (Figure 5c; light horizontal lines) and 2) they correspond to model levels. In the unforced experiment (left panels), the gradients in PV are fairly uniform between 10-60 degrees at 335 K but are concentrated over a relatively narrow latitude band at 326 K. In the forced experiment (right panels), PV decreases in the subtropics immediately after the heating is turned on at day 10 (particularly at 326 K), and hence the regions of largest PV gradients are pushed towards the pole. By day 100, the regions of largest PV gradients are centered notably poleward of their original locations on both isentropic surfaces. At 335 K the region of largest PV gradients is not only pushed polewards but is also compressed meridionally. In both the unforced and forced experiments, the largest equatorward eddy fluxes of PV (blue shading) closely follow the gradients in the PV field: i.e., as the gradients in PV are pushed polewards by the heating, so are the attendant eddy fluxes of PV.

The poleward shift in the eddy fluxes of PV shown in Figure 6 reflects a poleward shift in the latitude of largest wave drag in the vicinity of the subtropical tropopause. The eddy fluxes of PV evidently track the regions of largest PV gradients, and thus the response of the total wave driving near the tropopause seems consistent with a diffusive model of the eddy fluxes: i.e., tropical heating changes the gradients in PV near the tropopause level through its effects on static stability; the eddy fluxes of PV adjust in a manner consistent with the flux-gradient relationship.

5. Summary and Discussion

a) Summary
Tropical heating leads to a robust poleward contraction of the extratropical jets and their eddy fluxes of heat and PV in a range of numerical experiments. The experiments analyzed here reveal the following novel aspects of the eddy response to such heating in the dry dynamical core of a general circulation model:

1) In general, heating in the tropical troposphere directly affects the meridional slope of the isentropes in the subtropical and extratropical troposphere due to its projection on the climatological-mean isentropic surfaces. In the specific case considered here, the heating intersects lower tropospheric isentropic surfaces not in the deep tropics but in the subtropics. As such, the heating increases the isentropic slope in the extratropics and decreases the slope in the subtropics.

2) The changes in the isentropic slope are coincident with anomalous heat fluxes in the lower troposphere: the poleward eddy fluxes of heat are increased in regions where the isentropic slopes are anomalously steep, and decreased where the isentropic slopes are flattened. Thus, the poleward shift in the latitude of lower tropospheric eddy heat fluxes is consistent with the direct effect of the heating on the baroclinic stability of the extratropical troposphere.

3) At upper levels, the heating leads to low PV in the vicinity of the subtropical tropopause via its effects on atmospheric static stability. The anomalous eddy fluxes of PV (which are indicative of the wave forcing near the tropopause) are down the gradient of the anomalies in the PV field. Thus, the poleward shift in wave breaking at the tropopause is consistent with a diffusive model of the eddy fluxes of PV: regions where the pole-equator gradients in PV are increased are marked by enhanced equatorward eddy fluxes of PV, and vice versa.
The key results of this study are summarized in Figure 7. Panel a) shows the response to the heating in terms of the gradients in the PV field (contours) and the eddy fluxes of PV (shading) at 326 K; panel b) shows the response in terms of the isentropic slope at 310 K (contours) and the eddy fluxes of heat at 700 hPa (shading). The results in the top panel are derived by taking the difference between the gradients of the results in the lower panels of Figure 6; the results in the middle panel are reproduced from Figure 4b.

The anomalous eddy fluxes of PV (Figure 7a) and heat (Figure 7b) are both down the gradient of the anomalies in the zonal-mean circulation. At the tropopause, the decrease in PV is largest ~45 degrees (Figure 5c), thus the anomalous equator-pole PV gradient at the tropopause is negative in the subtropics but positive at middle latitudes (Figure 7a). The associated anomalous eddy fluxes of PV are down-gradient in both regions (Figure 7a). In the lower-middle troposphere, the anomalous eddy fluxes of heat are down-gradient in the sense that they are anomalously poleward where the baroclinicity is enhanced, and vice versa (Figure 7b).

The anomalous eddy fluxes of heat and PV shown in Figures 7a and 7b exhibit a high degree of vertical coherence, i.e., regions of anomalous poleward heat fluxes near the surface are overlain by regions of anomalous equatorward PV fluxes at the tropopause, and vice versa. As noted in Section 4, the strong degree of vertical coherence is consistent with the fact that the PV fluxes aloft are driven by the anomalous tropospheric heat fluxes as they damp at the tropopause level. Thus the anomalous eddy fluxes of heat and PV in Figure 7a & 7b do not arise independently. Rather, they are both a consequence of the changes in the heat fluxes within the free troposphere.
To see this more clearly, Figure 8 shows a schematic representation of the results in this paper. The top panel shows the response in terms of the eddy fluxes of heat and PV; the bottom panel shows the same response in terms of the anomalous EP fluxes and their divergences. The direct effect of the heating is to reduce PV at the tropopause (via a decrease in static stability), reduce the isentropic slope (reduce baroclinicity) in the subtropical troposphere, and increase the isentropic slope (increase baroclinicity) in the mid-latitude troposphere. The eddy response includes anomalous poleward eddy heat fluxes where the baroclinicity is increased, anomalous equatorward heat fluxes where the baroclinicity is decreased, and anomalous down-gradient eddy fluxes of PV at the tropopause.

When viewed solely from the perspective of the PV field at the tropopause, the anomalous eddy fluxes of PV can be interpreted as due to the in-situ anomalous PV gradients (Figure 8, top panel). However, when viewed from the perspective of the lower troposphere, the anomalous eddy fluxes of PV are clearly also due to the changes in tropospheric baroclinicity and thus the source of wave activity in the troposphere (Figure 8, bottom panel). That is: the changes in eddy heat fluxes within the troposphere give rise to anomalous convergence and divergence of the vertical component of the EP flux at the tropopause level. Presumably, both the source of wave activity within the troposphere and the gradients in PV at the tropopause play a role in determining the total changes in the eddy PV fluxes aloft.

\[ b) \ Discussion \]
As noted in Section 2, there are several caveats to the diffusive model when applied to the eddy fluxes of PV. Most importantly, radiation plays a first-order role in determining the surface gradients in temperature, but the eddies themselves play a first-order role in determining the PV gradients aloft (e.g., Held 1999). The key role of the eddies in setting the PV gradients in the free atmosphere complicates the interpretation of cause and effect between the eddies and the mean-flow there. Additionally, the eddy flux of PV consists of two primary terms: the vertical gradient in the eddy heat flux and the horizontal gradient in the eddy momentum flux. Thus, anomalies in the eddy fluxes of PV are not necessarily indicative of changes in the barotropic component of the flow (i.e., the component driven by the vertically integrated eddy momentum flux convergence).

In principle, the results could be used to provide a precise quantitative estimate of the changes in the eddy fluxes if: 1) we estimated the eddy diffusivities; and 2) the eddy diffusivities are assumed to remain fixed in response to the heating. However, the eddy diffusivities are extremely noisy when calculated directly from data (they are proportional to the meridional PV flux divided by the spatial gradients in PV). Additionally, there is no "a priori" reason to expect the diffusivities to remain fixed in time. Nevertheless, a diffusive model of the eddy fluxes of PV has been applied with apparent success towards problems such as the height of the tropopause (e.g., Schneider 2004), and the results shown here suggest that such a model provides an excellent qualitative approximation of the eddy response to diabatic heating at the tropopause level. The diffusive model also provides a qualitative approximation of the barotropic component of the flow. Here is why:
Theoretically, the eddy forcing that drives the barotropic component of the response can be quantified by vertically integrating the anomalous eddy fluxes of PV from the surface (where the PV flux is equivalent to the heat flux) to the top of the atmosphere. In practice, the barotropic component of the response can be estimated from the response in the lower tropospheric eddy heat flux. That is because the lower tropospheric eddy heat flux reflects the generation of wave activity near the surface. Assuming a component of the wave activity generated near the surface propagates meridionally in the free atmosphere, it follows that latitude bands marked by anomalously poleward heat fluxes at the surface are accompanied by anomalous eddy momentum flux convergence aloft, and vice versa (e.g., Robinson 1996; 2000). Thus, regions of anomalous poleward heat fluxes should coincide with anomalous eastward flow at the surface, and regions of anomalous equatorward heat fluxes should coincide with anomalous westward flow at the surface. The meridional coincidence between eddy heat fluxes and the surface flow has been used to explain the persistence of annular variability (e.g., Robinson 1996; 2000) and exploited recently by Lu et al. (2010) to explain the simulated surface wind response to changes in sea-surface temperatures.

Figure 7c provides support for the above line of reasoning. The shading shows the response of the eddy heat fluxes at 700 hPa (reproduced from the middle panel); the contours show the response of the zonal wind at 700 hPa. There is evidently a high degree of correspondence between the changes in the near-surface flow and the near-surface heat fluxes: regions of anomalous poleward surface heat fluxes are accompanied by anomalous eastward wind anomalies and thus - by inference - anomalous convergence
of the eddy momentum flux aloft; regions of anomalous equatorward surface heat fluxes are accompanied by anomalous westward wind anomalies.

The results in this study provide an alternative mechanism for interpreting the tropospheric eddy response to diabatic heating than that outlined in Chen and Held (2007) and Chen et al. (2008). Following the mechanism outlined in those studies, the response to tropical heating can be interpreted as follows: 1) the heating leads to eastward zonal wind anomalies at the extratropical tropopause via thermal wind; 2) the anomalies in the thermal wind lead to an increase in the eddy phase speed and thus drive a poleward shift in the latitude of largest wave breaking in the subtropics; and 3) the poleward shift in wave breaking in the subtropics induces a meridional circulation that perturbs baroclinicity and thus wave generation at lower levels. In contrast, the mechanism outlined here suggests that: 1) the poleward shift in wave generation at lower levels is driven fundamentally by the projection of the heating onto the isentropic surfaces at extratropical latitudes; and 2) the poleward shift in wave breaking near the tropopause is due to the poleward shift in the heat fluxes within the troposphere (which converge at the tropopause level) and the diffusive nature of the eddy fluxes of PV. Which mechanism is most applicable to the real atmosphere remains to be determined.

We applied a heating profile that spans the entire tropical troposphere and decreases monotonically along pressure surfaces (Figure 1). The heating projects onto the upward sloping isentropic surfaces at subtropical latitudes, and thus leads to a dipole in baroclinicity, with increased baroclinicity in the extratropics and decreased baroclinicity in the subtropics. The changes in the eddy fluxes of heat and PV will also adjust the baroclinicity as the integration proceeds. But as shown in Figure 4, the eddy response is
to first order consistent with the changes in baroclinicity forced directly by the diabatic heating.

Presumably, the changes in extratropical baroclinicity should be very different for heatings that do not intersect the upward sloping isentropic surfaces in the subtropics. But interestingly, dipolar changes in baroclinicity very similar to those shown in Figure 4 also arise for heatings that are confined to the deep tropics (i.e., as in Figure 2c of Butler et al. 2010; results not shown). The robustness of the dipole in baroclinicity to the meridional scale of the heating suggests that the weak horizontal temperature gradients in the tropical atmosphere (Charney 1963) place a strong constraint on the structure of the extratropical response: i.e., tropical heatings with both narrow and broad meridional scales yield nearly identical patterns of tropical temperature rises (Butler et al. 2010; c.f. Figure 2), and thus have very similar influences on the meridional slopes of the isentropic surfaces at subtropical latitudes. It is not immediately clear why the response to the warm phase of the El Niño-Southern Oscillation yields a seemingly opposite signed response to that shown here (Chen et al. 2010), albeit the absence of moisture in our experiments may a play a key role in this regard. We considered damping the decreases in subtropical baroclinicity by applying a heating that decreases monotonically along isentropic surfaces (i.e., rather than pressure surfaces). But in practice, the resulting heating profiles appear unphysical when projected onto pressure coordinates (e.g., consider a heating profile that follows the 310 K surface in Figure 1a).

We have applied a (macro)turbulence perspective to the extratropical eddy response to imposed tropical heating. In our view, the extratropical response is - to first order - driven by dry dynamical processes. However, moist processes must inevitably
play a key role in determining the true shape and amplitude of the tropical heating. It would be interesting to test the hypotheses outlined here in experiments forced not by idealized tropical heating, but by realistic changes in sea-surface temperatures in a climate model that includes moist processes.

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References


Figure Captions

Figure 1. (a) The time-mean, ensemble-mean potential temperature for the unforced experiment (contours) and the applied thermal forcing (shading) in pressure coordinates. Note that the contour interval changes from tropospheric to stratospheric levels. (b) The applied thermal forcing in isentropic coordinates. The grey shading indicates regions where the isentropes intersect the surface. Units are K/day for the forcing and K for the isentropic surfaces. The solid black line represents the height of the dynamic tropopause, defined as the contour where PV=2 PVU.

Figure 2. (a) The temperature response to the thermal forcing. Contour interval is 2 K. (b) The pressure response in isentropic coordinates. Contour interval is 40 hPa. (c) The response of the zonal wind to the thermal forcing. Contour interval is 2 m/s. (d) As in (c) but for isentropic coordinates. In all panels, the response is defined as the difference between the ensemble-mean of the forced experiment and the ensemble-mean of the unforced experiment. The response is averaged over days 100-150. The solid black line indicates the contour where PV=2 PVU. The zero line is omitted.

Figure 3. The response of the zonal wind as a function of time and latitude at (a) 350 K and (b) 700 hPa. The heating is turned on at day 10 (indicated by the solid vertical line). Contour interval is 2 m/s. The zero line is omitted.

Figure 4. (a) Contours: The projection of the thermal forcing onto the 310 K surface as a function of time and latitude. Contour interval is 0.005 K/day. Shading: The response of
pressure at 310 K (units of hPa). (b) Contours: The response of the isentropic slope at 310 K. Contour interval is 0.2 m/km ($m$ in the vertical direction/$km$ in the meridional direction). The zero line is omitted. Positive values denote increasing height with latitude. Shading: The response of the eddy heat flux at 700 hPa (units of K m/s).

**Figure 5.** Potential vorticity (contours) and the eddy flux of PV (shading) averaged over days 100-150 for (a) the unforced experiment, (b) the forced experiment, and (c) the response (i.e., the difference). The thin horizontal lines show 335 K and 326 K isentropes. The solid black lines in panels (a) and (b) correspond to the 2 PVU contour; the solid black lines in panel (c) reproduce the 2 PVU contour from the unforced experiment (lower line) and forced experiment (upper line). PV is given in units of PVU=10^{-6} m^2 K s^{-1} kg^{-1}. Eddy fluxes of PV are given in units of PVU m s^{-1}.

**Figure 6.** PV (contours) and the eddy flux of PV (shading) as a function of time and latitude at (top) 335 K and (bottom) 326 K. Left panels show results for the unforced experiment; right panels show results for the forced experiment. PV is given in units of PVU=10^{-6} m^2 K s^{-1} kg^{-1}. Eddy fluxes of PV are given in units of PVU m s^{-1}.

**Figure 7.** The response to the heating as a function of time and latitude. (a) Contours: The meridional gradient in PV at 326 K. Contour interval is .25x10^{-12} PVU/m. Shading: The eddy PV flux at 326 K. (b) Reproduced from Figure 4b. Contours: The response of the isentropic slope at 310 K. Positive values denote increasing height with latitude. Contour interval is 0.2 m/km ($m$ in the vertical direction/$km$ in the meridional direction).
Shading: The response of the eddy heat flux at 700 hPa. (c) Contours: The response of the zonal wind at 700 hPa. Contour interval is 2 m/s. The zero line is omitted. Shading: The response of the eddy heat flux at 700 hPa. The units of eddy PV fluxes is PVU m s⁻¹; the units of eddy heat fluxes is K m/s.

Figure 8. Schematic of the response to tropical tropospheric heating (indicated by the light gray shading) in terms of the anomalous eddy fluxes of heat and PV (top) and the anomalous EP fluxes and their divergences (bottom). The heating intersects the isentropic surface denoted by the dashed line in the subtropics. Thus, the heating increases (decreases) the isentropic slope (i.e., the baroclinicity) poleward (equatorward) of the region where the isentrope intersects the heating. At the same time, the reduction in static stability above the heating leads to low PV there. The heat fluxes are anomalously poleward where the baroclinicity is increased, and vice versa, whereas the anomalous PV fluxes are down-gradient as indicated. The down-gradient PV fluxes are consistent with a diffusive model of the eddy fluxes at the tropopause, as indicated in the top panel. But they are also mandated by the vertical derivative of the heat flux, which is indicated in terms of the anomalous EP flux in the bottom panel. See text for details.
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