

AT 655  
Objective Analysis in the Atmospheric Sciences

Prepare a 1-2 paragraph 'mini' proposal for your term project. The proposal does not have to be perfect; it is meant simply as a letter of intent. You can send this to me via email. (Due March 20, 2007.)

**Homework 4 (due April 6, 2007):**

Summarize and critique the following paper:

Quadrelli, R., and J. M. Wallace, 2004: A simplified linear framework for interpreting patterns of Northern Hemisphere wintertime climate variability. *J. Climate*, **17**, 3728-3744.

(Your summary and critique should be maximum 2 pages single spaced. Spend no more than 1 page summarizing the paper and spend at least 1 page critiquing the paper.)

**Homework 5 (due April 6, 2007):**

1. Calculate the EOFs and PCs of the following 4x2 data matrix by hand. Following the convention used in class, the sampling domain lies in the columns.

$$A = \begin{bmatrix} 2 & 4 \\ -3 & -6 \\ 1 & -2 \\ 0 & 4 \end{bmatrix}$$

a) First calculate  $A^T A$ . Explain why it is more efficient to eigenanalyze  $A^T A$  than  $AA^T$  in this case.

b) Find the eigenvalues and eigenvectors of  $A^T A$  (i.e., the EOFs of  $A$ ). Scale the eigenvectors to length one.

c) Use the relationship  $A = U\Sigma V^T$  to find the PC time series that correspond to the EOFs found in b), above.

*Hints:*

- Since  $A = U\Sigma V^T$ , it follows that  $AV = U\Sigma$ . Hence, the  $i$ th PC can be found by solving  $Av_i = u_i\sigma_i$ .

- Remember: the singular values in the SVD are the square roots of the eigenvalues of  $A^T A$ .

d) List the orthonormal EOFs and PCs of  $A$ , and the variance explained by each EOF/PC pair.

e) Demonstrate that  $A = U\Sigma V^T$  is satisfied.

2. Use EOF analysis to extract the dominant patterns of variability in the 500 hPa height field during winter. (Note: Use the 500 hPa height data available on the class website).

a) Preparing the data:

- Remove the seasonal cycle in mean 500 hPa height from each grid point.
- Reduce the 500 hPa dataset to December, January, February monthly-mean values.
- Since the grid size decreases as you move towards the pole, weight each grid box (i.e., multiply the time series at each grid box) by the square root of the cosine of latitude (the weights are based on the square root of the cosine so that the covariance matrix is weighted by the cosine of latitude).
- Note: do not remove the spatial mean from the data.

b) Calculate the first three PCs of the data (i.e., the PC time series corresponding to the 3 largest eigenvalues of the dispersion matrix). You can use either SVD or eigenanalysis of the dispersion matrix. If you use eigenanalysis of the dispersion matrix, I would suggest

eigenanalyzing the spatial dispersion matrix (in this case,  $AA^T$ , where  $A$  is the weighted 500 hPa height anomaly dataset with the sampling domain in the column space.  $A$  should be size 129x1008, and  $AA^T$  size 129x129).

c) Regress the unweighted 500 hPa anomaly height data (DJF values only) onto standardized values of the 3 leading PCs found in b). You should have one regression map for each PC. Note the resulting maps are similar to the corresponding EOFs, but not identical: the EOFs lie in the weighted state space; the regression maps do not.

d) Plot the spectrum of the first 10 eigenvalues (scaled as % variance explained), with error bars.

e) Briefly discuss your results. Compare your findings with those presented in Quadrelli and Wallace (from HW4). How do the first 2 EOFs of 500 hPa height relate to the first two EOFs of SLP (as documented in Quadrelli and Wallace)?